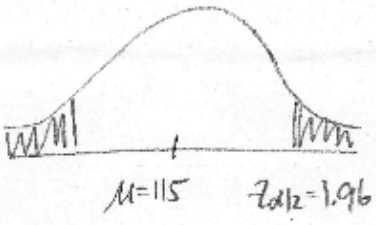
Math 254 Assignment Three

Name:_____

Due: In class on Thursday September 8.
Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [4 marks] We are testing $H_0 : \mu = 115$ versus $H_a : \mu \neq 115$ at significance level $\alpha = 0.05$ with a sample of size 64. We are given $\sigma = 10$. What is the probability of not rejecting H_0 if the true value of μ is 113.5?

Find non-rejection region



$$115 \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$115 \pm 1.96 \times \frac{10}{\sqrt{64}}$$

$$\boxed{112.55 \leq \bar{x} \leq 117.45}$$

Now find:

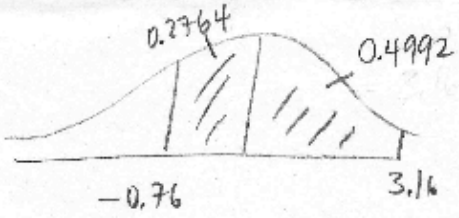
$$P(112.55 \leq \bar{x} \leq 117.45 \mid \mu = 113.5)$$

$$= P(-0.76 \leq \bar{x} \leq 3.16)$$

$$z = \frac{112.55 - 113.5}{10/\sqrt{64}} = -0.76$$

$$z = \frac{117.45 - 113.5}{10/\sqrt{64}} = 3.16$$

=



$$= 0.7756$$

2. [8 marks] Times for the 100 m freestyle swim are recorded for two swimmers on 12 different days. Both swimmers' times are normally distributed for this event. Test with $\alpha = 0.05$ whether Swimmer 1 is faster than Swimmer 2.

Swimmer 1: 59.21 59.84 59.36 60.04 60.11 59.57
Swimmer 2: 59.77 59.52 60.21 60.36 60.05 60.00

Independent Samples

a) State H_0 and H_a

(3) $H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 < 0$

b) What are the assumptions?

both populations are normal

n_1 or $n_2 < 30$

$\frac{\text{larger } s^2}{\text{smaller } s^2} \leq 3$

c) Do you reject H_0 or not? Show all your work.

(2) $\bar{x}_1 \approx 59.6883$ $\bar{x}_2 = 59.985$

$s_1^2 \approx 0.1350$ $s_2^2 \approx 0.0916$

$n_1 = 6$ $n_2 = 6$

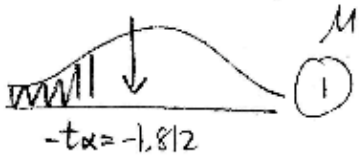
Small sample use t

$$t = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$s^2 = \frac{s_1^2 + s_2^2}{10} \approx 0.1133$$ (1)

≈ -1.527 $df = 10$

Don't reject H_0 $\mu_1 = \mu_2$



(1)

d) What can you say about the p-value of the test?

(1) $df = 10$

$\alpha = 0.10$ $\alpha = 0.05$

1.372 ↑ 1.812

t

$0.05 < p < 0.10$

3. [7 marks] An experimenter weighs a single object and gets the readings below (in kg). The readings for this object are normally distributed. Test at $\alpha = 0.05$ whether the scale has variance $\sigma^2 = 0.005$ kg^2.

1.115 1.284 1.281 1.348

a) State H_0 and H_a

$H_0: \sigma^2 = 0.005$ $H_a: \sigma^2 \neq 0.005$ two-tailed (2)

b) What are the assumptions?

normal population (1)

c) Do you reject H_0 or not? Show all your work.

$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$ $s^2 \approx 0.0099$ (1)

$= \frac{3s^2}{0.005}$

≈ 5.94 (1)

$\alpha = 0.05$
$df = 3$
$\chi^2_{0.025} = 9.348$
$\chi^2_{0.975} = 0.216$

Don't reject H_0 (1)
$\sigma^2 = 0.005$

d) What can you say about the p-value of the test? (1)

Most correct answer:

	$\alpha = 0.90$		$\alpha = 0.1$
$df = 3$	0.584	↑	6.251

$0.1 < p < 0.9$

[Since $\chi^2 \geq 0$ don't need to double here.]

Also acceptable:
$0.2 < p < 1.8$
since I taught you to double for all two-tailed tests.
Simplify to: $0.2 < p$

4. [6 marks] A national survey states that 67% of college students are under the age of 25, 21% are between 25 and 30, 8% are between 30 and 40, and 4% are over 40. A random sample of Camosun College students yielded the data below. Test at $\alpha = 0.05$ whether the age distribution at Camosun agrees with the national survey.

Age:	< 25	$25 - 30$	$30 - 40$	> 40
Frequency	138	62	32	18

a) State H_0 and H_a

H_0: $p_1 = 0.67$ $p_2 = 0.21$ $p_3 = 0.08$ $p_4 = 0.04$

H_a: at least one proportion differs (2)

b) Do you reject H_0 or not? Show all your work.

Goodness-of-Fit

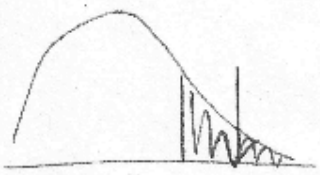
$\chi^2 = \sum \frac{(O-E)^2}{E}$

≈ 20.515 (1)

$n = 138 + 62 + 32 + 18 = 250$

$E_i = np_i$

O	138	62	32	18
(2) E	167.5	52.5	20	10



$\chi^2_\alpha = 7.815$

χ^2 test is always right-tailed

$\alpha = 0.05$ $df = 3$ #categories -1

Reject H_0 (1)

At least one proportion differs.