Stat 254 Assignment Three

Name:_____

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [3 marks] You are given the following measurements taken from a normal population. Find a 90% upper confidence bound for the population mean. Round your answer to three decimal places.

82.295 85.307 94.860 82.668 87.431

$n < 30$
σ unknown
normal population
} use t rather than z

90% UCB for μ:

$\bar{x} + t_\alpha \cdot \frac{s}{\sqrt{n}}$

$= 86.5122 + 1.533\left(\frac{5.11237}{\sqrt{5}}\right)$

≈ 90.017

$\bar{x} = 86.5122$
$s \approx 5.11237$
$n = 5$
$1-\alpha = 0.9 \quad \alpha = 0.1$
$df = n-1 = 4$
$t_{0.1} = 1.533$

$\mu \leq 90.017$

2. [7 marks] Below are commute times (in minutes) for two different students on 12 different days. Both students' commute times are normally distributed. Test at $\alpha = 0.05$ whether Student 1 has a shorter average commute than Student 2.

Student 1:	37	40	42	46	47	52
Student 2:	41	46	41	45	49	54

left-tailed test

a) State H_0 and H_a

$H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 < 0$

b) State any necessary assumptions

n_1 or $n_2 < 30$, both populations normal and $\frac{\text{larger } s^2}{\text{smaller } s^2} \leq 3$

c) Do you reject H_0 or not? Show all your work.

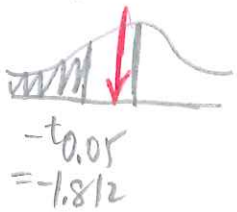
$\bar{x}_1 = 44$ $s_1^2 = 29.2$ $n_1 = 6$ $\bar{x}_2 = 46$ $s_2^2 = 24.8$ $n_2 = 6$

$$t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \quad s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = 27$$

$$= \frac{44 - 46 - 0}{\sqrt{27\left(\frac{1}{6} + \frac{1}{6}\right)}}$$

$$\approx -0.667$$

$df = n_1 + n_2 - 2 = 10$



Don't reject H_0
$\mu_1 \approx \mu_2$

d) What can you say about the p-value of the test?

	$t_{0.1}$	$t_{0.05}$
$df = 10$	1.372	1.812

$|t|$

$p > 0.1$

(Don't double for one-tailed tests)

3. [5 marks] An experimenter weighs a single object repeatedly and gets a set of masses (in mg) that are normally distributed. Below is a random sample of the masses. Find a 95% confidence interval for the standard deviation of the masses. Round your values to 3 decimal places.

1.189 1.303 1.317 1.196

Start with a 95% CI for σ^2:

$$\sigma^2 = \frac{(n-1)s^2}{\chi^2}$$

$n = 4$

$s^2 \approx 0.00464$

$df = n - 1 = 3$

$\chi^2_{0.025} = 9.348$

$\chi^2_{0.975} = 0.216$

$$\sigma_1^2 = \frac{3(0.00464)}{9.348}$$

$$\approx 0.00149$$

$$\sigma_2^2 = \frac{3(0.00464)}{0.216}$$

$$\approx 0.06444$$

$$0.00149 \leq \sigma^2 \leq 0.06444$$

Taking square roots:

$$\boxed{0.039 \leq \sigma \leq 0.254}$$