~~Math~~ Stat 254 Assignment Two

Name:______________________________

**Assignments must be completed on this paper. Marks may be deducted for not showing all your work.**

1. [3 marks] An old plane has four engines which operate independently. Each engine operates correctly on 89% of flights. Find the probability that at least two of the four engines operate correctly on the plane's next flight. Round your answer to three decimal places.

Binomial X= # engines that operate correctly (1)

$n=4 \quad p=0.89 \quad q=0.11$

$P(X\geq 2) = P(X=2)+P(X=3)+P(X=4)$ (1)

$= {}_4C_2(0.89)^2(0.11)^2 + {}_4C_3(0.89)^3(0.11) + {}_4C_4(0.89)^4$ (1)

$\approx 0.995$

or $P(X\geq 2) = 1-P(X=1)-P(X=0)$

(-1) if you forgot $P(X=0)$

2. [3 marks] The average number of accidents per week on a busy highway is 1.5. Find the probability distribution for the number of accidents over the next two weeks. Round your probabilities to two decimal places. Ignore any probabilities that are less than 1%.

Poission $\frac{1.5 \text{ accidents}}{\text{week}} = \frac{3 \text{ accidents}}{2 \text{ weeks}}$

$\boxed{\text{Use } \mu=3}$

$P(X=k)=\frac{3^k e^{-3}}{k!}$

| X | P(X=k) |
|---|---|
| 0 | 0.05 |
| 1 | 0.15 |
| 2 | 0.22 |
| 3 | 0.22 |
| 4 | 0.17 |
| 5 | 0.10 |
| 6 | 0.05 |
| 7 | 0.02 |

3. [4 marks] Find the expected value and the variance for the random variable $X$ with the following probability density function: $f(x) = \frac{1}{(5\ln 2)x}$ for $4 \le x \le 128$ and $f(x) = 0$ otherwise. Keep the expected value in exact form, but round the variance to two decimal places.

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \cancel{\int_{-\infty}^{4} 0\,dx} + \int_{4}^{128} \frac{1}{5\ln 2}\,dx + \cancel{\int_{128}^{\infty} 0\,dx}$$

$$= \left[\frac{x}{5\ln 2}\right]_4^{128}$$

$$= \frac{124}{5\ln 2}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_{4}^{128} \frac{x}{5\ln 2}\,dx$$

$$= \left[\frac{x^2}{10\ln 2}\right]_4^{128}$$

$$= \frac{16368}{10\ln 2}$$

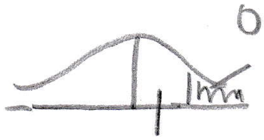
Now $\sigma^2 = E(X^2) - [E(X)]^2$

$$= \frac{16368}{10\ln 2} - \left[\frac{124}{5\ln 2}\right]^2$$

$$\approx 1081.28$$

4. [4 marks] The volume placed in a bottle by a bottling machine is normally distributed with mean $\mu$ and SD $\sigma$. Over a long period of time it is observed that 6.43% of bottles contain more than 2.2364 L, and 3.07% of bottles contain more than 2.2609 L. Find $\mu$ and $\sigma$.

0.0643

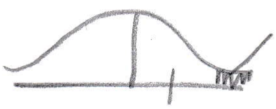


0.4357

$z = 1.52$

$X = 2.2364$

0.0307



0.4693

$z = 1.87$

$X = 2.2609$

$z = \frac{X-\mu}{\sigma}$ produces two equations:

① $1.52 = \frac{2.2364-\mu}{\sigma}$

or $\mu + 1.52\sigma = 2.2364$ ③

② $1.87 = \frac{2.2609-\mu}{\sigma}$

$\mu + 1.87\sigma = 2.2609$ ④

④ − ③: $0.35\sigma = 0.0245$

$\boxed{\sigma = 0.07}$

$\sigma = 0.07 \rightarrow$ ③: $\mu + 1.52(0.07) = 2.2364$

$\boxed{\mu = 2.13}$

$\boxed{\mu = 2.13\,L \qquad \sigma = 0.07\,L}$

5. [3 marks] As part of their job interview process, 2000 aspiring engineers write a standardized test which is scored out of 100. The mean test score is 72 with a variance of 36. A random sample of 80 tests is selected. Find the probability that the mean of the sampled test scores is less than 73.

$\mu = 72 \quad \sigma^2 = 36 \quad n = 80$

$\sigma = 6$

$(n \geq 30 \checkmark)$

$z = \dfrac{X - \mu}{\sigma}$ for individual measurements

$z = \dfrac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$ for samples with multiple measurements

$P(\bar{x} < 73)$

$$z = \frac{\bar{x} - \mu}{(\sigma/\sqrt{n})} = \frac{73 - 72}{(6/\sqrt{80})} \approx 1.49$$

$= P(z < 1.49)$

$=$ 


$= 0.9319$

6. [2 marks] A certain website tracks how long visitors stay on the site. A sample of 75 visitors stayed for an average of 1.17 minutes, with a standard deviation of 0.38 minutes. Find a 95% lower confidence bound for the average amount of time visitors stay on the site. Round your answer to two decimal places.

$n = 75 \quad \bar{x} = 1.17 \quad s = 0.38$

$(n \geq 30 \checkmark)$

95% LCB for $\mu$:

$1 - \alpha = 0.95$

$z_\alpha = 1.645$

$$\bar{x} - z_\alpha \frac{s}{\sqrt{n}}$$

$$1.17 - 1.645\left(\frac{0.38}{\sqrt{75}}\right)$$

$\approx 1.10$

$\boxed{\mu \geq 1.10 \text{ min}}$

7. [6 marks] Test whether the population proportions $p_1$ and $p_2$ are equal at the 10% significance level given the following sample data: $n_1 = 1000, \hat{p}_1 = 0.81, n_2 = 600, \hat{p}_2 = 0.85$.

(-1) if you used $\hat{p}_1, \hat{p}_2$ here

a) State $H_0$ and $H_a$

(1) $H_0: p_1 = p_2$ $\qquad H_a: p_1 \neq p_2$ $\qquad$ 2-tailed

b) Check any necessary assumptions.

(1) $n_1\hat{p}_1,\ n_1\hat{q}_1,\ n_2\hat{p}_2,\ n_2\hat{q}_2$ all $> 5$ ✓ $\qquad \hat{q}_1 = 0.19 \quad \hat{q}_2 = 0.15$

c) Do you reject $H_0$? Show all your work.

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$ $\qquad x = $ # that have the property

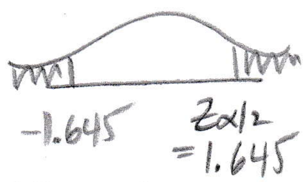
$$\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

$\hat{p} = 0.825$

$\hat{q} = 0.175$ $\qquad$ (1)

$$= \frac{0.81 - 0.85}{\sqrt{(0.825)(0.175)\left(\frac{1}{1000} + \frac{1}{600}\right)}}$$

(1) $\approx -2.04$

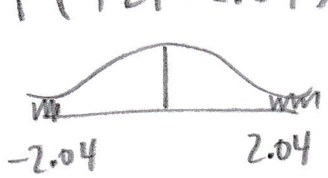



$\boxed{\text{Reject } H_0 \quad p_1 \neq p_2}$ (1)

d) Find the $p$-value.

$p = P(|z| > 2.04)$

=




$= 2(0.5 - 0.4793)$ $\qquad$ (1)

$= 0.0414$

8. [5 marks] Find the probability of making a Type II error in the hypothesis test below if the true value of $\mu$ is 76.5.

Test $H_0$: $\mu = 77$ at $\alpha = 0.05$ with $\overline{x} = 76.8, s = 1.3, n = 60$.

Find the probability that we don't reject Ho even though Ho is false.

Non-rejection region:

$$\mu_0 \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$77 \pm 1.960 \left( \frac{1.3}{\sqrt{60}} \right)$$

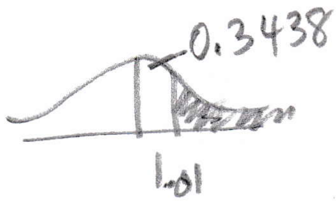
2

$$76.67 \le \bar{x} \le 77.33$$

Now find $P(76.67 \le \bar{x} \le 77.33)$ using $\mu = 76.5$

$$= P(1.01 \le z \le 4.95)$$

$=$




$$= 0.5 - 0.3438$$

$$= 0.1562$$ 1

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$z_1 = \frac{76.67 - 76.5}{(1.3/\sqrt{60})} \approx 1.01$$

$$z_2 = \frac{77.33 - 76.5}{(1.3/\sqrt{60})} \approx 4.95$$

2