

Math 251 Assignment Two

Name: _____

Due: In class on Monday August 22

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [3 marks] In a certain textbook the mean number of typos on each page is 0.8. What is the probability of finding more than two typos on the 20th page of the textbook? Round your answer to 2 decimal places.

Poisson
 $\mu = 0.8$

$$\begin{aligned} P(X > 2) &= 1 - P(X=0) - P(X=1) - P(X=2) \\ &= 1 - \left(\frac{0.8^0}{0!} + \frac{0.8^1}{1!} + \frac{0.8^2}{2!} \right) e^{-0.8} \\ &\approx 0.05 \\ &\quad \uparrow \uparrow \end{aligned}$$

Round to 2 decimal
places!

2. [4 marks] A fair die is rolled 300 times. What is the approximate probability of observing between 40 and 62 fours? Round your answer to 2 decimal places.

$$\text{Binomial } n=300 \quad p=\frac{1}{6} \quad q=1-p=\frac{5}{6}$$

Since $np > 5$ ✓ $nq > 5$ ✓
we can approximate by

normal curve with $\mu=np=50$

$$\sigma = \sqrt{npq} = \sqrt{\frac{1500}{36}}$$

$$P(40 \leq X \leq 62) \underset{\text{binomial}}{\approx} P(39.5 \leq X \leq 62.5) \underset{\text{normal}}{\approx}$$

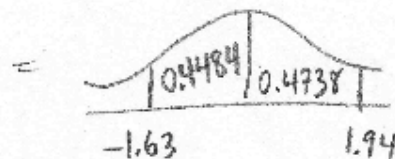
$$z = \frac{39.5 - 50}{\sqrt{\frac{1500}{36}}}$$

$$\approx -1.63$$

$$z = \frac{62.5 - 50}{\sqrt{\frac{1500}{36}}}$$

$$\approx 1.94$$

$$= P(-1.63 \leq z \leq 1.94)$$



$$= 0.9222$$

$$\approx 0.92$$

The approximate probability
is 0.92

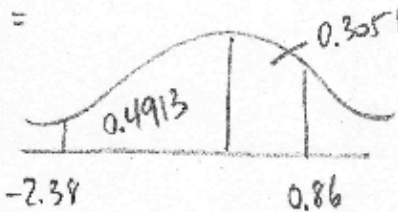
3. [4 marks] 31% of all Canadians are basketball fans. A group of 100 Canadians are polled on whether they are basketball fans.

a) What is the probability that the sample proportion is between 20% and 35%? Round your answer to 2 decimal places.

$$p = 0.31 \quad n = 100$$

$$P(0.2 \leq \hat{p} \leq 0.35)$$

$$= P(-2.38 \leq z \leq 0.86) \quad (1)$$



$$= 0.7964$$

$$\approx 0.80 \quad (1)$$

Central Limit
Theorem

$$z = \frac{0.2 - p}{\sqrt{\frac{pq}{n}}} \quad (1)$$

$$= \frac{0.2 - 0.31}{\sqrt{\frac{0.31 \times 0.69}{100}}}$$

$$= \frac{0.2 - 0.31}{\sqrt{\frac{0.31 \times 0.69}{100}}}$$

$$\approx -2.38$$

$$\approx -2.38$$

$$z = \frac{0.35 - p}{\sqrt{\frac{pq}{n}}}$$

$$\approx 0.86$$

$$\approx 0.86$$

b) What assumption(s) do you have to check?

$$np > 5, \quad nq > 5 \quad \checkmark \quad (1)$$

4. [4 marks] A sample of 600 drill bits shows that 7 are defective.

a) Find a 98% confidence interval for the proportion defective. Round your answer to 3 decimal places.

$$\hat{p} = \frac{7}{600} \quad n = 600 \quad \hat{q} = 1 - \hat{p} = \frac{593}{600}$$

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \quad (1)$$

$$\frac{7}{600} \pm 2.326 \sqrt{\frac{7 \times 593}{600^2}} \quad (1)$$

(1)

$$0.001167 \leq p \leq 0.022167$$

Round to 3 decimal places.

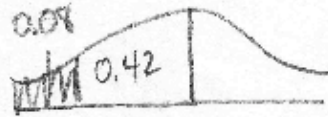
b) Would a 90% confidence interval for p have a longer or shorter length than your 98% confidence interval? Explain.

A 90% CI would have a
(1) shorter length than the 98% CI
since we are less certain that p
is contained in the interval.

Or: because $z_{\alpha/2}$ is smaller for
 $1 - \alpha = 0.9$ than for $1 - \alpha = 0.98$

5. [4 marks] X is normally distributed with mean 90 and SD $\sigma = 12$.

a) Find the 8th percentile for X . Round your answer to 2 decimal places.



$$z = -1.41$$

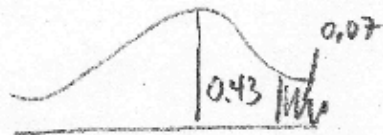


Reverse look-up
area 0.42

$$\frac{X - 90}{12} = -1.41$$

$$X = 73.08$$

b) Find the 93rd percentile for X . Round your answer to 2 decimal places.



$$z = 1.48$$



Reverse look-up
area 0.43

$$\frac{X - 90}{12} = 1.48$$

$$X = 107.76$$

6. [6 marks] At a bottling plant 35 cans are sampled and found to have mean volume 352.12 mL and sample SD 5.40 mL. Test at $\alpha = 0.02$ whether the mean volume of all cans at the plant is less than 355 mL.

a) State H_0 and H_a

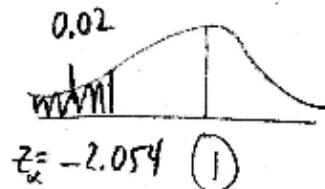
$$H_0: \mu = 355 \quad H_a: \mu < 355 \quad (1)$$

b) What assumption(s) do you need to check?

$$n \geq 30 \quad (1)$$

c) Do you reject H_0 or not? Show all your work.

$$\begin{aligned} z &= \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \\ &= \frac{352.12 - 355}{5.4/\sqrt{35}} \\ &\approx -3.16 \quad (1) \end{aligned}$$



Reject H_0 . (1)
 $\mu < 355$

d) Find the p-value of the test.

$$\begin{aligned} p &= P(z < -3.16) \\ &= \text{Area under curve to the left of } -3.16 \\ &= 0.5 - 0.4992 \\ &= 0.0008 \quad (1) \end{aligned}$$