Stat

Math 254 Assignment One

Name:_____

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [3 marks] The data set below represents house prices in a certain city.

Price ($)	Relative Frequency
400,000	0.1
600,000	0.45
800,000	0.25
1,000,000	0.15
1,200,000	0.05

a) Calculate the mean and the median.

$\mu = (400{,}000)(0.1) + \ldots + (1{,}200{,}000)(0.05) = 720{,}000$

median = 600,000

b) Based on the mean and the median, in which direction is the data skewed?

median < mean | Skewed to the right

2. [4 marks] A data set has 1200 measurements, a mean of 65, and a standard deviation of 7. What can you say about the number of measurements that are less than 37?

$37 = \mu - k\sigma$

$37 = 65 - k(7)$

$k = 4$

$\geq 1 - \frac{1}{k^2}$ of data lies in $\mu \pm k\sigma$

$\geq \frac{15}{16}$ of data lies in $[37, 93]$

$\leq \frac{1}{16}$ of data lies outside $[37, 93]$

$\leq \frac{1}{16}$ of data is less than 37

≤ 75 measurements are less than 37

2

2

3. [2 marks] Construct a data set with two measurements which has a mean of 83 and a population variance of 16. Show your work.

Let the two measurements be x and y.

$$\frac{x+y}{2} = 83 \quad (1)$$

$$\frac{(x-83)^2+(y-83)^2}{2} = 16 \quad (2)$$

(1): $x = 166 - y \rightarrow (2)$

$$\frac{(166-y-83)^2 + (y-83)^2}{2} = 16$$

$$(83-y)^2 + (y-83)^2 = 32$$

$$y^2 - 166y + 6889 + y^2 - 166y + 6889 = 32$$

$$2y^2 - 332y + 13746 = 0$$

$$2(y^2 - 166y + 6873) = 0$$

$$y^2 - 166y + 6873 = 0$$

$$y = \frac{166 \pm \sqrt{(-166)^2 - 4(1)(6873)}}{2}$$

$$y = 87, 79$$

Whichever y-value you choose, the data set is $\boxed{79, 87}$

4. [3 marks] If the data has a linear relationship, find the equation of the least squares regression line. Round your values to one decimal place. If the data does not have a linear relationship, write *nonlinear*.

a)

x	1.1	1.6	0.8	0.7	1.9	1.2
y	-2.5	-2.7	-5.6	-5.1	-2.2	-4.9

$|r| \geq 0.7 \Rightarrow$ linear

$r \approx 0.8$ linear ✓ $\quad y \approx 2.6x - 7.0$

b)

x	3.4	3.7	2.9	2.6	2.7	2.2
y	8.2	8.5	7.5	5.1	8.3	7.1

$r \approx 0.6$ non-linear

5. [4 marks] A lab has two smoke alarms. 98.8% of fires are detected by Alarm A; 98.4% of fires are detected by Alarm B; 0.7% of fires are detected by Alarm A but not Alarm B. Find the probability that a fire is detected by:

a) both alarms

Make a table or Venn diagram

	A	not A
B		
not B	0.007	

	A	not A
B	0.981	
not B	0.007	

	A	not A
B	0.981	0.003
not B	0.007	

	A	not A
B	0.981	0.003
not B	0.007	0.009

$P(\text{both}) = 0.981$

b) at least one alarm

$P(\text{at least one}) = 0.007 + 0.981 + 0.003 = 0.991$

c) neither alarm

$P(\text{neither}) = 0.009$

6. [3 marks] A coin is tossed eight times. Find the probability that the first three tosses are heads or the last toss is tails. Round your answer to two decimal places.

$n(S) = 2 \times 2 \times \ldots = 2^8 = 256$

Recall $n(A \text{ or } B) = n(A) + n(B) - n(\text{both})$

$\#\text{ways} = \#\,HHH\_\_\_\_\_ + \#\,\_\_\_\_\_T - \#\,HHH\_\_\_\_T$

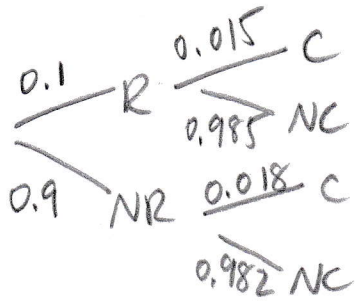
$= 2^5 + 2^7 - 2^4$

$= 144$

$\text{probability} = \frac{144}{256} \approx 0.56$

7. [3 marks] Ten percent of all cars are red. Of the red cars, 1.5% are convertibles. Of the non-red cars, 1.8% are convertibles. Find:

a) The probability that a red car is not a convertible



$P(NC|R) = 0.985$

b) The probability that a convertible is not red

$$P(NR|C) = \frac{P(NR \text{ and } C)}{P(C)}$$

$$= \frac{0.9(0.018)}{[0.1(0.015) + 0.9(0.018)]}$$

$$\approx 0.915$$

8. [3 marks] 12 deep-water and 18 shallow-water samples are drawn from the inner harbour. Four samples are randomly selected for further testing. Let X be the number of deep-water samples selected.

a) Find the probability distribution of X. Round the probabilities to two decimal places.

X = # deep	outcome	# ways	$P(X)$
0	0D and 4S	$18C4$	0.11
1	1D and 3S	$12C1 \cdot 18C3$	0.36
2	2D and 2S	$12C2 \cdot 18C2$	0.37
3	3D and 1S	$12C3 \cdot 18C1$	0.14
4	4D and 0S	$12C4$	0.02

(1) total = 27,405 (1)

More concisely:

X	$P(X)$
0	0.11
1	0.36
2	0.37
3	0.14
4	0.02

b) Find the expected value of X

$E(X) = 0(0.11) + 1(0.36) + \ldots + 4(0.02)$ (1)

$= 1.6$