(1) $\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$

$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2!}{s^3}\right\}$

$= \frac{1}{2}t^2$

(2) $\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{48}{s^5}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1!}{s^2} - 2\left(\frac{4!}{s^5}\right)\right\}$

$= t - 2t^4$

(3) $\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s-2}\right\}$

$= t - 1 + e^{2t}$

(4) $\mathcal{L}^{-1}\left\{\frac{4s}{4s^2+1}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+\frac{1}{4}}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+\left(\frac{1}{2}\right)^2}\right\}$

$= \cos\frac{t}{2}$

(5) $\mathcal{L}^{-1}\left\{ \frac{2s-6}{s^2+9} \right\}$

$= \mathcal{L}^{-1}\left\{ 2\left(\frac{s}{s^2+9}\right) - 2\left(\frac{3}{s^2+9}\right) \right\}$

$= 2\cos 3t - 2\sin 3t$

(6) $\mathcal{L}^{-1}\left\{ \frac{1}{s^2+3s} \right\}$

$= \mathcal{L}^{-1}\left\{ \frac{1}{s(s+3)} \right\}$

$$\frac{1}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$$

$$1 = A(s+3) + Bs$$

Sub $s=0$: $1 = 3A \Rightarrow A = \frac{1}{3}$

Sub $s=-3$: $1 = -3B \Rightarrow B = \frac{-1}{3}$

$= \mathcal{L}^{-1}\left\{ \frac{1}{3}\cdot\frac{1}{s} - \frac{1}{3}\frac{1}{s+3} \right\}$

$= \frac{1}{3} - \frac{1}{3}e^{-3t}$

(7) $\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s-3}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{(s+3)(s-1)}\right\}$

$$\frac{s}{(s+3)(s-1)} = \frac{A}{s+3} + \frac{B}{s-1}$$

$$s = A(s-1) + B(s+3)$$

Sub $s=1$: $1 = 4B \Rightarrow B = \frac{1}{4}$

$s=-3$: $-3 = -4A \Rightarrow A = \frac{3}{4}$

$= \mathcal{L}^{-1}\left\{\frac{3}{4}\frac{1}{s+3} + \frac{1}{4}\frac{1}{s-1}\right\}$

$= \frac{3}{4}e^{-3t} + \frac{1}{4}e^{t}$

(8) $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s-3)(s-6)}\right\}$

$$\frac{s}{(s-2)(s-3)(s-6)} = \frac{A}{s-2} + \frac{B}{s-3} + \frac{C}{s-6}$$

$$s = A(s-3)(s-6) + B(s-2)(s-6) + C(s-2)(s-3)$$

Sub $s=2$: $2 = 4A \Rightarrow A = \frac{1}{2}$

$s=3$: $3 = -3B \Rightarrow B = -1$

$s=6$: $6 = 12C \Rightarrow C = \frac{1}{2}$

$= \mathcal{L}^{-1}\left\{\frac{1}{2}\frac{1}{s-2} - \frac{1}{s-3} + \frac{1}{2}\frac{1}{s-6}\right\}$

$= \frac{1}{2}e^{2t} - e^{3t} + \frac{1}{2}e^{6t}$

Section 7.2

(9) $\mathcal{L}^{-1}\left\{\frac{1}{s^3+5s}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s(s^2+5)}\right\}$

$$\frac{1}{s(s^2+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+5}$$

$1 = A(s^2+5) + (Bs+C)s$

Sub $s=0$: $1 = 5A \Rightarrow A = \frac{1}{5}$

s^2 coefficient: $0 = A + B$

$\Rightarrow B = -\frac{1}{5}$

s coefficient: $0 = C$

$= \mathcal{L}^{-1}\left\{\frac{1}{5}\frac{1}{s} - \frac{1}{5}\frac{s}{s^2+5}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{5}\frac{1}{s} - \frac{1}{5}\frac{s}{s^2+\sqrt{5}^2}\right\}$

$= \frac{1}{5} - \frac{1}{5}\cos\sqrt{5}\,t$

Section 7.2

(10) $\mathcal{L}^{-1}\left\{\dfrac{2s-4}{(s^2+s)(s^2+1)}\right\}$

$= \mathcal{L}^{-1}\left\{\dfrac{2s-4}{s(s+1)(s^2+1)}\right\}$

$$\frac{2s-4}{s(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2+1}$$

$$2s-4 = A(s+1)(s^2+1) + Bs(s^2+1) + (Cs+D)s(s+1)$$

Sub $s=0$: $-4 = A$

$s=-1$: $-6 = -2B \Rightarrow B = 3$

s^3 coefficient: $0 = A+B+C$
$\Rightarrow C = 1$

Can sub any s-value to get the last coefficient.

Sub $s=1$: $-2 = A(2)(2) + B(2) + (C+D)(2)$

$-2 = 4A + 2B + 2C + 2D$

$-2 = -16 + 6 + 2 + 2D$

$6 = 2D$

$D = 3$

$= \mathcal{L}^{-1}\left\{-4\dfrac{1}{s} + 3\dfrac{1}{s+1} + \dfrac{s}{s^2+1} + \dfrac{3}{s^2+1}\right\}$

$= -4 + 3e^{-t} + \cos t + 3\sin t$

(11) $\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)(s^2+4)}\right\}$

$$\frac{1}{(s^2+1)(s^2+4)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4}$$

$$1 = (As+B)(s^2+4) + (Cs+D)(s^2+1)$$

Sub $s=i$: $1 = (Ai+B)(3)$

$1 = 3Ai + 3B$

$A = 0$

$B = \frac{1}{3}$

s^3 coefficient : $0 = A + C$

$C = 0$

constant coefficient : $1 = 4B + D$

$1 = \frac{4}{3} + D$

$D = \frac{-1}{3}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{3}\frac{1}{s^2+1} - \frac{1}{3}\frac{1}{s^2+4}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{3}\frac{1}{s^2+1} - \frac{1}{6}\frac{2}{s^2+4}\right\}$$

$$= \frac{1}{3}\sin t - \frac{1}{6}\sin 2t$$

(12) $y' - y = 1 \quad , \quad y(0) = 0$

1) Apply $\mathcal{L}$

$$sY(s) - y(0) - Y(s) = \frac{1}{s}$$

$$sY(s) - Y(s) = \frac{1}{s}$$

2) Solve for $Y(s)$

$$(s-1)Y(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s(s-1)}$$

3) Apply $\mathcal{L}^{-1}$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\}$$

$$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$1 = A(s-1) + Bs$$

Sub $s = 0$: $\quad 1 = -A \Rightarrow A = -1$

$s = 1$: $\quad 1 = B$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{-1}{s} + \frac{1}{s-1}\right\}$$

$$y(t) = -1 + e^{t}$$

or $y = -1 + e^{t}$

(13) $y' + 6y = e^{4t}, \quad y(0) = 2$

1) Apply $\mathcal{L}$

$$sY(s) - y(0) + 6Y(s) = \frac{1}{s-4}$$

$$sY(s) - 2 + 6Y(s) = \frac{1}{s-4}$$

2) Solve for $Y(s)$

$$(s+6)Y(s) = 2 + \frac{1}{s-4}$$

$$Y(s) = \frac{2}{s+6} + \frac{1}{(s-4)(s+6)}$$

3) Apply $\mathcal{L}^{-1}$

$$y(t) = 2e^{-6t} + \mathcal{L}^{-1}\left\{\frac{1}{(s-4)(s+6)}\right\}$$

$$\frac{1}{(s-4)(s+6)} = \frac{A}{s-4} + \frac{B}{s+6}$$

$$1 = A(s+6) + B(s-4)$$

Sub $s = -6$: $\quad 1 = -10B \Rightarrow B = -\frac{1}{10}$

$s = 4$: $\quad 1 = 10A \Rightarrow A = \frac{1}{10}$

$$y(t) = 2e^{-6t} + \mathcal{L}^{-1}\left\{\frac{1}{10}\frac{1}{s-4} - \frac{1}{10}\frac{1}{s+6}\right\}$$

$$= 2e^{-6t} + \frac{1}{10}e^{4t} - \frac{1}{10}e^{-6t}$$

$$= \frac{19}{10}e^{-6t} + \frac{1}{10}e^{4t}$$

Section 7.2

(14) $y'' + 5y' + 4y = 0$

$y(0) = 1, \quad y'(0) = 0$

1) Apply $\mathcal{L}$

$$s^2 Y(s) - s y(0) - y'(0) + 5[sY(s) - y(0)] + 4Y(s) = 0$$

$$s^2 Y(s) - s + 5sY(s) - 5 + 4Y(s) = 0$$

2) Solve for $Y(s)$: $(s^2 + 5s + 4)Y(s) = s + 5$

$$Y(s) = \frac{s+5}{(s^2+5s+4)}$$

3) Apply $\mathcal{L}^{-1}$:

$$y(t) = \mathcal{L}^{-1}\left\{\frac{s+5}{s^2+5s+4}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s+5}{(s+1)(s+4)}\right\}$$

$$\frac{s+5}{(s+1)(s+4)} = \frac{A}{s+1} + \frac{B}{s+4}$$

$$s+5 = A(s+4) + B(s+1)$$

Sub $s = -4$: $1 = -3B \Rightarrow B = \frac{-1}{3}$

$s = -1$: $4 = 3A \Rightarrow A = \frac{4}{3}$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{4}{3}\frac{1}{s+1} - \frac{1}{3}\frac{1}{s+4}\right\}$$

$$y(t) = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}$$

(15) $y'' + y = \sqrt{2}\sin\sqrt{2}t$

$y(0) = 10, \quad y'(0) = 0$

1) Apply $\mathcal{L}$

$$s^2 Y(s) - sy(0) - y'(0) + Y(s) = \sqrt{2}\left(\frac{\sqrt{2}}{s^2+2}\right)$$

$$s^2 Y(s) - 10s + Y(s) = \frac{2}{s^2+2}$$

2) Solve for $Y(s)$

$$(s^2+1)Y(s) = 10s + \frac{2}{s^2+2}$$

$$Y(s) = \frac{10s}{s^2+1} + \frac{2}{(s^2+1)(s^2+2)}$$

3) Apply $\mathcal{L}^{-1}$

$$y(t) = 10\cos t + \mathcal{L}^{-1}\left\{\frac{2}{(s^2+1)(s^2+2)}\right\}$$

$$\frac{2}{(s^2+1)(s^2+2)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+2}$$

$$2 = (As+B)(s^2+2) + (Cs+D)(s^2+1)$$

Sub $s = i$: $2 = (Ai+B)(1)$

$2 = Ai + B$

$A = 0, \quad B = 2$

s^3 coefficient: $0 = A + C$

$C = 0$

Constant coefficient: $2 = 2B + D$

$2 = 4 + D$

$D = -2$

$\rightarrow$

15 Cont'd

$$y(t) = 10\cos t + \mathcal{L}^{-1}\left\{\frac{2}{s^2+1} - \frac{2}{s^2+2}\right\}$$

$$= 10\cos t + \mathcal{L}^{-1}\left\{\frac{2}{s^2+1} - \sqrt{2}\left(\frac{\sqrt{2}}{s^2+\sqrt{2}^2}\right)\right\}$$

$$= 10\cos t + 2\sin t - \sqrt{2}\sin\sqrt{2}t$$