Section 1.2

(1) $$y = \frac{1}{1+C_1 e^{-x}}$$

$x = 0$
$y = -\frac{1}{3}$: $$-\frac{1}{3} = \frac{1}{1+C_1}$$

$$-3 = 1 + C_1$$

$$C_1 = -4$$

$$y = \frac{1}{1-4e^{-x}}$$

(3) $$y = \frac{1}{x^2+C}$$

$x = 2$
$y = \frac{1}{3}$: $$\frac{1}{3} = \frac{1}{4+C}$$

$$3 = 4 + C$$

$$C = -1$$

$$y = \frac{1}{x^2-1}$$

Interval of solution: $x^2 - 1 \neq 0$
$x^2 \neq 1$
$x \neq \pm 1$

Possible Intervals: $x < -1$ or $-1 < x < 1$ or $x > 1$

But interval must contain $x = 2$

Interval of Solution is $1 < x < \infty$

$$\boxed{\begin{aligned}\cos\tfrac{\pi}{6} &= \tfrac{\sqrt{3}}{2}\\ \sin\tfrac{\pi}{6} &= \tfrac{1}{2}\end{aligned}}$$

9 $x = C_1\cos t + C_2\sin t$

$t=\frac{\pi}{6}$, $x=\frac{1}{2}$: $\quad \frac{1}{2} = C_1\frac{\sqrt{3}}{2} + C_2\left(\frac{1}{2}\right)$

Multiply by 2:

$1 = \sqrt{3}C_1 + C_2$ Call this 1

$x' = -C_1\sin t + C_2\cos t$

$t=\frac{\pi}{6}$, $x'=0$: $\quad 0 = -C_1\left(\frac{1}{2}\right) + C_2\left(\frac{\sqrt{3}}{2}\right)$

Multiply by 2:

$0 = -C_1 + \sqrt{3}C_2$ Call this 2

Eliminate C_1:

1 : $\quad 1 = \sqrt{3}C_1 + C_2$

$\sqrt{3}\times$2 : $\quad 0 = -\sqrt{3}C_1 + 3C_2$

$+$

$1 = 4C_2$

$C_2 = \frac{1}{4}$

$C_2 = \frac{1}{4} \rightarrow$ 2 : $\quad 0 = -C_1 + \frac{\sqrt{3}}{4}$

$C_1 = +\frac{\sqrt{3}}{4}$

$$x = \frac{\sqrt{3}}{4}\cos t + \frac{1}{4}\sin t$$

(11) $y = C_1 e^{x} + C_2 e^{-x}$

$x=0$, $y=1$: $1 = C_1 + C_2$ (1)

$y' = C_1 e^{x} - C_2 e^{-x}$

$x=0$, $y'=2$: $2 = C_1 - C_2$ (2)

(1) + (2): $3 = 2C_1$

$C_1 = \frac{3}{2}$

$C_1 = \frac{3}{2} \to$ (1): $1 = \frac{3}{2} + C_2$

$C_2 = \frac{-1}{2}$

$y = \frac{3}{2} e^{x} - \frac{1}{2} e^{-x}$

(25) Let $y' = f$

Check if f and $\frac{\partial f}{\partial y}$ are both continuous near the given point.

$$f = \sqrt{y^2 - 9}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}(y^2-9)^{-1/2}(2y)$$

$$= \frac{y}{\sqrt{y^2-9}}$$

Near the point $(1,4)$

$\Rightarrow y \approx 4$

$\Rightarrow$ Both f and $\frac{\partial f}{\partial y}$ are continuous.

Yes the IVP has a unique solution.

(27) Same as (25) but with a different point.

Near the point $(2,-3)$

$\Rightarrow y \approx -3$

<u>It's not true</u> that f and $\frac{\partial f}{\partial y}$ are both continuous near the given point.

The IVP is <u>not guaranteed</u> to have a unique solution.