

## Complex Numbers Exercises

**Answers appear on the next page.**

- Express  $z = (1 + 2i)(4 - 6i)^2$  in the form  $a + bi$ .
- Find  $\bar{z}$  and  $|z|$  for:
  - $z = 2 + 7i$
  - $z = -3 - 5i$
- Find  $\frac{1}{z}$  for:
  - $z = 1 - 5i$
  - $z = 1 + i$
- Find  $\frac{z_1}{z_2}$  for  $z_1 = 2 + i$  and  $z_2 = -7 + 5i$ .
- Let  $z_1 = 2 + 2\sqrt{3}i$  and  $z_2 = \frac{\sqrt{3}}{3} + \frac{1}{3}i$ .
  - Convert  $z_1$  and  $z_2$  to polar form.
  - Compute  $z_1 z_2$
  - Compute  $\frac{z_1}{z_2}$ .
- Compute  $(i + 1)^8$  by converting  $i + 1$  to polar form.
- Solve  $z^6 - 64 = 0$ .
- Let  $w = a + bi$  and  $z = c + di$ . Show that  $|wz| = |w||z|$ .

## ANSWERS

1.  $z = 76 - 88i$

2. a)  $2 - 7i, \sqrt{53}$

b)  $-3 + 5i, \sqrt{34}$

3. a)  $\frac{1}{26} + \frac{5}{26}i$

b)  $\frac{1}{2} - \frac{1}{2}i$

4.  $\frac{-9}{74} - \frac{17}{74}i.$

5. a)  $z_1 = 4[\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3})]$  and  $z_2 = \frac{2}{3}[\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6})].$

b)  $\frac{8}{3}[\cos(\frac{\pi}{2}) + i \sin(\frac{\pi}{2})] = \frac{8}{3}i.$

c)  $6[\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6})].$

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7.  $z = \pm 2, 1 \pm \sqrt{3}i, -1 \pm \sqrt{3}i.$

8.

$$\begin{aligned} |wz| &= |(ac - bd) + (ad + bc)i| \\ &= \sqrt{(ac - bd)^2 + (ad + bc)^2} \\ &= \sqrt{[(ac)^2 - 2acbd + (bd)^2] + [(ad)^2 + 2adbc + (bc)^2]} \\ &= \sqrt{(ac)^2 + (bd)^2 + (ad)^2 + (bc)^2} \\ &= \sqrt{(a^2 + b^2)c^2 + (a^2 + b^2)d^2} \\ &= \sqrt{(a^2 + b^2)(c^2 + d^2)} \\ &= \sqrt{a^2 + b^2} \sqrt{c^2 + d^2} \\ &= |w||z| \end{aligned}$$