

Math 251 X02 Assignment Three

Name: _____

Due: In class on Tuesday August 30

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [4 marks] Find $\det A$ using Gaussian Elimination.

$$A = \begin{bmatrix} 2 & 1 & 6 & 9 \\ 2 & 1 & 3 & 0 \\ 6 & 8 & 1 & 0 \\ 4 & 12 & 1 & 1 \end{bmatrix}$$

2. [6 marks] Given $A = \begin{bmatrix} 2 & 1 & 1 & 6 \\ 0 & 4 & 2 & 0 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

a) Find the characteristic polynomial of A .

b) Find a basis for the eigenspace E_2 .

c) Explain why A is not diagonalizable.

3. [7 marks] Find the matrix P that diagonalizes $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 6 \\ 0 & 0 & -3 \end{bmatrix}$.

Show all your work.

4. [3 marks] Use the method involving the adjoint to find A^{-1} .

$$A = \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 2 \\ 1 & 3 & 6 \end{bmatrix}$$

5. [2 marks] Let $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right\}$ such that $x = 5t, y = -2t, z = 10t$.

Find a basis for W^\perp .

6. [3 marks] $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}, \begin{bmatrix} 7 \\ 7 \\ 4 \end{bmatrix}, \begin{bmatrix} -69 \\ 45 \\ 42 \end{bmatrix} \right\}$ is an orthogonal basis for \mathbb{R}^3 .

Find a vector \mathbf{u} in \mathbb{R}^3 such that:

$$\mathbf{u} \cdot \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix} = -150, \quad \mathbf{u} \cdot \begin{bmatrix} 7 \\ 7 \\ 4 \end{bmatrix} = 342 \quad \text{and} \quad \mathbf{u} \cdot \begin{bmatrix} -69 \\ 45 \\ 42 \end{bmatrix} = 2850.$$