Math 251 X01 Assignment Two

Name:____________________________

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [6 marks] Find the general form of span$\left(\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right)$.

Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = c_1 \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ 1

$$\left[\begin{array}{cc|c} 1 & 1 & a \\ 1 & 2 & b \\ 1 & 3 & c \\ 3 & 4 & d \end{array}\right]$$

$$\begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \\ R_4 - 3R_1 \end{array} \quad \left[\begin{array}{cc|c} 1 & 1 & a \\ 0 & 1 & b-a \\ 0 & 2 & c-a \\ 0 & 1 & d-3a \end{array}\right]$$

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$$\begin{array}{l} R_3 - 2R_2 \\ R_4 - R_2 \end{array} \quad \left[\begin{array}{cc|c} 1 & 1 & a \\ 0 & 1 & b-a \\ 0 & 0 & a-2b+c \\ 0 & 0 & -2a-b+d \end{array}\right] \quad \begin{array}{l} c-a-2(b-a) \\ = c-a-2b+2a \\ = a-2b+c \end{array} \quad \begin{array}{l} d-3a-(b-a) \\ = d-3a-b+a \\ = -2a-b+d \end{array}$$

System is consistent when:

$a-2b+c=0 \rightarrow c=-a+2b$

and

$-2a-b+d=0 \rightarrow d=2a+b$

Span $= \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ such that } c=-a+2b \text{ and } d=2a+b \right\}$

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$= \left\{ \begin{bmatrix} a & b \\ -a+2b & 2a+b \end{bmatrix} \right\}$

2. [6 marks] Write $A = \begin{bmatrix} 1 & -2 \\ 5 & 8 \end{bmatrix}$ and A^{-1} as a product of elementary matrices.

$$\begin{bmatrix} 1 & -2 \\ 5 & 8 \end{bmatrix}$$

$R_2 - 5R_1$ $\begin{bmatrix} 1 & -2 \\ 0 & 18 \end{bmatrix}$ $\quad E_1 = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$

$R_2/18$ $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ $\quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{18} \end{bmatrix}$

$R_1 + 2R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad E_3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$$\underbrace{E_3 E_2 E_1}_{A^{-1}} A = I$$

So $A^{-1} = E_3 E_2 E_1$

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and $A = (A^{-1})^{-1}$

$= E_1^{-1} E_2^{-1} E_3^{-1}$

$= \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 18 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

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3. [4 marks] Consider the set S and the statement below. If the statement is true, prove it. If it's false, give a counterexample.

$$S = \{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ such that } x = 9z \text{ and } y = -7z \}$$

If $\mathbf{u}$ and $\mathbf{v}$ are in S, then $\mathbf{u}+\mathbf{v}$ is in S.

$S = \{ \begin{bmatrix} 9z \\ -7z \\ z \end{bmatrix}$ where z is a real number$\}$

Proof: Let $\vec{u}$ and $\vec{v}$ be in S

$\Rightarrow \vec{u} = \begin{bmatrix} 9a \\ -7a \\ a \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 9b \\ -7b \\ b \end{bmatrix}$ (2)

(for some real numbers a and b)

$\Rightarrow \vec{u}+\vec{v} = \begin{bmatrix} 9a+9b \\ -7a-7b \\ a+b \end{bmatrix}$

$= \begin{bmatrix} 9(a+b) \\ -7(a+b) \\ a+b \end{bmatrix}$ (1)

$\Rightarrow \vec{u}+\vec{v}$ is in S because it has the correct form. (1)

Alternatively: $S = \{ z \begin{bmatrix} 9 \\ -7 \\ 1 \end{bmatrix} \}$

And go through a similar proof.

4. [4 marks] Find a basis for span([1, 3, 2, 3], [2, 6, 5, 7], [−1, −3, 2, 1], [1, 3, 4, 5]) consisting of some of the original vectors.

Let $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 6 & -3 & 3 \\ 2 & 5 & 2 & 4 \\ 3 & 7 & 1 & 5 \end{bmatrix}$ 1

Find a basis for Column space of A

$$\begin{array}{r} \\ R_2-3R_1 \\ R_3-2R_1 \\ R_4-3R_1 \end{array} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 2 \\ 0 & 1 & 4 & 2 \end{bmatrix}$$

$$\begin{array}{r} \\ R_2 \leftrightarrow R_4 \\ \\ \\ \end{array} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & 2 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{r} \\ R_3-R_2 \\ \\ \\ \end{array} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ 2

Columns 1 & 2 of A

Basis $= \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 5 \\ 7 \end{bmatrix} \right\}$ 1

or {[1, 3, 2, 3], [2, 6, 5, 7]}