Math 251 X02 Assignment Two

Name:____________________________________

**Due: In class on Tuesday August 9**

Assignments must be completed on this paper. Marks may be deducted for not showing all your work.

1. [5 marks] a) Find the inverse of the matrix below using Gauss-Jordan Elimination.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 8 & -6 & -4 \\ -4 & 3 & 1 \end{bmatrix}$$

b) Use part a) to solve the system below.

$$\begin{aligned} x + 2y + 5z &= 5 \\ 8x - 6y - 4z &= -70 \\ -4x + 3y + z &= 33 \end{aligned}$$

2. [3 marks] A square matrix $A$ is called **skew symmetric** if $A^T = -A$. Show that if $B$ is an $n \times n$ matrix then $C = B - B^T$ is skew symmetric.

3. [4 marks] Find the LU factorization of the matrix below.

$$A = \begin{bmatrix} 2 & 0 & 1 & 1 \\ -8 & 1 & 0 & 3 \\ 6 & 3 & 1 & 10 \\ 4 & -2 & 2 & 0 \end{bmatrix}$$

4. [3 marks] Use Question 3 to solve the system below.

$$\begin{aligned} 2x_1 \qquad\quad + \quad x_3 + \quad x_4 &= 1 \\ -8x_1 + x_2 \qquad\quad + 3x_4 &= 19 \\ 6x_1 + 3x_2 + x_3 + 10x_4 &= 30 \\ 4x_1 - 2x_2 + 2x_3 \qquad\quad &= -12 \end{aligned}$$

5. [3 marks] Show that the set $S$ of vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that $x = 4y$ and $z = -7y$ is a subspace of $\mathbb{R}^3$.

6. [4 marks] Find a basis for the row space of $A$ consisting of rows of $A$.

$$A = \begin{bmatrix} 1 & 2 & -1 & 9 \\ -2 & -3 & 1 & 6 \\ -1 & 1 & -2 & 63 \\ 4 & 3 & 3 & 3 \end{bmatrix}$$

7. [3 marks] Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 12 \\ 13 \\ -6 \end{bmatrix}$.

Find $T(\mathbf{v})$ for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ with:

$$T(\mathbf{v}_1) = \begin{bmatrix} -6 \\ 30 \\ 24 \\ 8 \end{bmatrix}, T(\mathbf{v}_2) = \begin{bmatrix} -2 \\ 8 \\ 6 \\ 2 \end{bmatrix} \text{ and } T(\mathbf{v}_3) = \begin{bmatrix} -1 \\ 9 \\ 6 \\ 0 \end{bmatrix}.$$