Math 251 X02
Test One

Time: 50 minutes
Total: 25 marks

Name: _____

1. [6 marks] Let $\mathbf{u} = \begin{bmatrix} 3 \\ -5 \\ 7 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -6 \\ -2 \\ 3 \end{bmatrix}$.

a) Find $||5\mathbf{u} - 4\mathbf{v}||$.

b) Calculate the angle between $\mathbf{u}$ and $\mathbf{v}$.

2. [4 marks] Find all vectors with length 19 that are perpendicular to both $\mathbf{u} = \begin{bmatrix} -20 \\ 9 \\ 31 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 7 \\ -4 \\ -7 \end{bmatrix}$.

3. [5 marks] Let $\mathbf{u} = [x, 2, -2], \mathbf{v} = [-3, 6, 1]$ and $\mathbf{w} = [9, -2, 3]$.

a) Find the volume of the parallelepiped determined by $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$. Your answer will involve x.

b) For what value(s) of x do $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ lie in a common plane?

4. [5 marks] Solve using Gauss-Jordan Elimination:

$$\begin{aligned} 2x - 4y + 30z &= 46 \\ 3x - 5y + 39z &= 59 \\ 7x - 11y + 87z &= 131 \end{aligned}$$

5. [5 marks] Find the distance between the point $P = (2, -3, 7)$ and the plane $4x - 7y + 11z = 12$.