# 2.2 Solving Systems

**Ex:** Solve by Gaussian Elimination

$$\begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} 3 & 4 & 5 & 41 \\ 0 & 2 & 1 & 16 \\ 0 & 8 & 7 & 76 \end{array}\right] \end{array}$$

Matrix → REF
Solve by Backsubstitution

**REF:** 1) Any zero rows @ bottom
2) Leading nonzero entries in each row move down and right

$$\begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} \textcircled{3} & 4 & 5 & 41 \\ 0 & \textcircled{2} & 1 & 16 \\ 0 & 0 & \textcircled{3} & 12 \end{array}\right] \end{array} \quad R_3 - 4R_2 \quad \text{REF}$$

$$3z = 12 \rightarrow \boxed{z = 4}$$

$$2y + z = 16 \rightarrow 2y + 4 = 16 \rightarrow \boxed{y = 6}$$

$$3x + 4y + 5z = 41 \rightarrow 3x + 24 + 20 = 41 \rightarrow \boxed{x = -1}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \\ 4 \end{bmatrix}$$

Ex: Solve by Gauss-Jordan elimination

$$\begin{matrix} x & y & z & \\ \end{matrix}\begin{bmatrix} 4 & 2 & 2 & 8 \\ 3 & 5 & 6 & 11 \\ 2 & 2 & 3 & 4 \end{bmatrix}$$

Matrix → RREF

RREF: 1) REF
2) Leading nonzero entry in each row is 1
3) Leading 1's have 0's everywhere else in their columns

$$\frac{R_1}{4}\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 3 & 5 & 6 & 11 \\ 2 & 2 & 3 & 4 \end{bmatrix}$$

$$\begin{matrix} R_2 - 3R_1 \\ R_3 - 2R_1 \end{matrix}\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & \frac{7}{2} & \frac{9}{2} & 5 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

$\frac{12}{2} - \frac{3}{2}$

current row − k(pivot row)

$$R_2 \leftrightarrow R_3 \quad \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & 1 & 2 & 0 \\ 0 & \frac{7}{2} & \frac{9}{2} & 5 \end{bmatrix}$$

$$\begin{matrix} R_1 - \frac{1}{2}R_2 \\ R_3 - \frac{7}{2}R_2 \end{matrix}\begin{bmatrix} 1 & 0 & -\frac{1}{2} & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -\frac{5}{2} & 5 \end{bmatrix}$$

$\frac{9}{2} - \frac{14}{2}$

current row − k(pivot row)

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} & 2 \end{bmatrix}$$

$R_3 \times \left(\frac{-2}{5}\right)$ $\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -2 \end{array}\right]$

$R_1 + \frac{1}{2}R_3$
$R_2 - 2R_3$

$\left[\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right]$ RREF

$x = 1$
$y = 4$
$z = -2$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$

Ex: Solve by Gauss-Jordan Elimination

$\left[\begin{array}{ccc|c} x & y & z & \\ 1 & 2 & 4 & -9 \\ 3 & 6 & 14 & -25 \\ 4 & 8 & 6 & -46 \end{array}\right]$

$R_2 - 3R_1$
$R_3 - 4R_1$

$\left[\begin{array}{ccc|c} 1 & 2 & 4 & -9 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & -10 & -10 \end{array}\right]$

$\frac{R_2}{2}$ $\left[\begin{array}{ccc|c} 1 & 2 & 4 & -9 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -10 & -10 \end{array}\right]$

$R_1 - 4R_2$

$R_3 + 10R_2$

$$\begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} \textcircled{1} & 2 & 0 & -13 \\ 0 & 0 & \textcircled{1} & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

RREF

- Circle the leading nonzero entry in each row
- Any column without a circle gets a parameter

$$\boxed{y = t}$$

$$x + 2y = -13 \rightarrow x = -13 - 2y \rightarrow \boxed{x = -13 - 2t}$$

$$\boxed{z = 1}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

Test 1 : $1.1 - 1.4$ $2.1 - 2.2$ (1.4 = Cross Product)