3.1 Matrix Operations

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 \\ 3 & 6 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$D = \begin{bmatrix} 7 & 8 \end{bmatrix}$

Find:

a) size of DC

$(1 \times 2)(2 \times 3)$

equal ✓

DC is 1×3

b) BC

$\begin{bmatrix} 2 & -1 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$[2 \ -1] \cdot \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} -2 & -1 & 0 \\ 27 & 36 & 45 \end{bmatrix}$

c) IC

$= C$

Recall $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d) A^T

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

e) $2A^T - 3B$

$= 2\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} - 3\begin{bmatrix} 2 & -1 \\ 3 & 6 \end{bmatrix}$

$= \begin{bmatrix} -4 & 9 \end{bmatrix}$

$$= \begin{bmatrix} -4 & 9 \\ -5 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Ex: Write as $A\vec{x} = \vec{b}$

$$\begin{cases} x - 2y = 17 \\ 11y = 23 \end{cases}$$

$$\begin{bmatrix} 1 & -2 \\ 0 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 17 \\ 23 \end{bmatrix}$$

Coefficients — Variables (column) — Constants (column)

Ex: Let $A = \begin{bmatrix} 8 & 2 \\ 12 & 3 \end{bmatrix}$

Find matrices B and C so that:

$$B \neq C$$

and $$AB = AC$$

$$AB = \begin{bmatrix} 8 & 2 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 12 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 8 & 2 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 12 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

Key Fact: $AB = AC \not\Rightarrow B = C$

Ex: Solve using a matrix multiplication

Production Line I makes 8 bikes and 7 skateboards per hour.

" II " 12 " 15 "

Production Line I runs for 4 hours

" II " 3 "

How many bikes and skateboards are produced?

$$\begin{matrix} & I & II \\ B \\ S \end{matrix}\begin{bmatrix} 8 & 12 \\ 7 & 15 \end{bmatrix} \qquad \begin{matrix} I & II \end{matrix}\begin{bmatrix} 4 & 3 \end{bmatrix}$$

items/hour hours

Method 1:

$$\begin{matrix} & I & II \\ B \\ S \end{matrix}\begin{bmatrix} 8 & 12 \\ 7 & 15 \end{bmatrix} \begin{matrix} I \\ II \end{matrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{matrix} B \\ S \end{matrix}\begin{bmatrix} 68 \\ 73 \end{bmatrix}$$

Method 2:

$$\begin{matrix} I & II \end{matrix}\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{matrix} I \\ II \end{matrix}\begin{bmatrix} 8 & 7 \\ 12 & 15 \end{bmatrix} = \begin{matrix} B & S \end{matrix}\begin{bmatrix} 68 & 73 \end{bmatrix}$$

Ex: $B = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

Find B^{195}

$$B^2 = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$B^3 = BB^2 = \frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix} -2 & -2 \\ 2 & -2 \end{bmatrix}$$

$$B^4 = BB^3 = \frac{1}{4}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -2 & -2 \\ 2 & -2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

⋮

$$B^8 = B^4 B^4 = (-I)(-I) = I$$

$$\boxed{B^8 = I}$$

$$8 \overline{)195} \quad 24$$
$$-16$$
$$35$$
$$-32$$
$$3 \leftarrow \text{remainder}$$

$$B^{195} = B^8 B^8 \cdots B^8 \cdot B^3$$
$$= I I \cdots I B^3$$
$$= B^3$$
$$= \frac{1}{2\sqrt{2}} \begin{bmatrix} -2 & -2 \\ 2 & -2 \end{bmatrix}$$

$$\boxed{\begin{aligned} IA &= A \\ II &= I \end{aligned}}$$