1. [5 marks] Find all the critical points for the following function.
Give your answer(s) in the form (x, y).
$z = 6x^4 - 16x^3 + 12x^2 + 3y^4 + 4y^3$

$$\left.\begin{aligned} z_x &= 24x^3 - 48x^2 + 24x \\ z_y &= 12y^3 + 12y^2 \end{aligned}\right\} \text{ both 0 or undefined}$$

AND

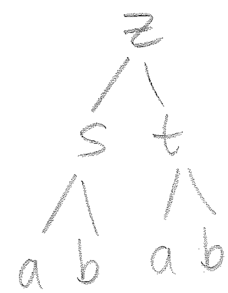
$24x^3 - 48x^2 + 24x = 0$	$12y^3 + 12y^2 = 0$
$24x(x^2 - 2x + 1) = 0$	$12y^2(y+1) = 0$
$24x(x-1)^2 = 0$	$y = 0$ or $y = -1$
$x = 0$ or $x = 1$	

There are 4 possibilities:

$\boxed{(0,0),\ (0,-1),\ (1,0),\ (1,-1)}$

2. [4 marks] Use the Multivariable Chain Rule to find z_b at $(a, b) = (1, 1)$ given:
$z = 2s^2 - 3st + 4t^2, \quad s = 2a^2 - 3b^2, \quad t = a^2 + b^2.$

$$z_b = z_s s_b + z_t t_b$$



$$z_b = (4s - 3t)(-6b) + (-3s + 8t)(2b)$$

At $(a,b) = (1,1)$, $(s,t) = (-1, 2)$

$$z_b \Big|_{(a,b)=(1,1)} = (-10)(-6) + (19)(2)$$

$$= 98$$

3. [4 marks] Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ given:

$f = 5\ln(x^3 - y^4) + 2e^{xy} + 3\sin(x^3y^3) - 4\cos(9x - 8y)$

$$\frac{\partial f}{\partial x} = 5 \frac{1}{x^3-y^4}(3x^2) + 2ye^{xy}$$

$$+ 9x^2y^3\cos(x^3y^3) + 36\sin(9x-8y)$$

$$= \frac{15x^2}{x^3-y^4} + 2ye^{xy} + 9x^2y^3\cos x^3y^3 + 36\sin(9x-8y)$$

$$\frac{\partial f}{\partial y} = 5 \frac{1}{x^3-y^4}(-4y^3) + 2xe^{xy} + 9x^3y^2\cos(x^3y^3)$$

$$- 32\sin(9x-8y)$$

$$= \frac{-20y^3}{x^3-y^4} + 2xe^{xy} + 9x^3y^2\cos x^3y^3 - 32\sin(9x-8y)$$

4. [5 marks] Find the equation of the tangent plane to
$z = \sqrt{50 - x^2 - y^2}$ at the point on the surface where $x = 3$ and $y = -4$.

$$z_x = \frac{1}{2}\left(50 - x^2 - y^2\right)^{1/2}(-2x)$$

$$= \frac{-x}{\sqrt{50-x^2-y^2}}$$

$$z_y = \frac{-y}{\sqrt{50-x^2-y^2}}$$

$$\vec{n} = [-z_x, -z_y, 1]$$

$$= \left[\frac{x}{\sqrt{50-x^2-y^2}}, \frac{y}{\sqrt{50-x^2-y^2}}, 1\right]$$

$$= \left[\frac{3}{5}, \frac{-4}{5}, 1\right]$$

$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$$

$$\begin{bmatrix} 3/5 \\ -4/5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3/5 \\ -4/5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$$

$z = \sqrt{50 - x^2 - y^2}$

or $\frac{3}{5}x - \frac{4}{5}y + z = 10$

or $3x - 4y + 5z = 50$

5. [5 marks] Let $f = \frac{3(\sqrt{x})}{y^4}$.
The maximum relative error in x is $\pm 4\%$.
The maximum relative error in y is $\pm 7\%$.
Estimate the maximum relative error in f.

$$f = 3x^{1/2} y^{-4}$$

$$f_x = \frac{3}{2} x^{-1/2} y^{-4} = \frac{3}{2\sqrt{x}\, y^4}$$

$$f_y = -12 x^{1/2} y^{-5} = \frac{-12\sqrt{x}}{y^5}$$

$$df = f_x\, dx + f_y\, dy$$

$$= \frac{3}{2\sqrt{x}\, y^4}\, dx - \frac{12\sqrt{x}}{y^5}\, dy$$

$$\frac{df}{f} = \frac{3}{2\sqrt{x}\, y^4}\left(\frac{y^4}{3\sqrt{x}}\right)dx - \frac{12\sqrt{x}}{y^5}\left(\frac{y^4}{3\sqrt{x}}\right)dy$$

$$= \frac{1}{2}\frac{dx}{x} - 4\frac{dy}{y}$$

To maximize this, use $\frac{dx}{x} = 4\%$ and $\frac{dy}{y} = -7\%$.

$$\left(\frac{df}{f}\right)_{max} = \frac{1}{2}(4\%) - 4(-7\%)$$

$$= 30\%$$