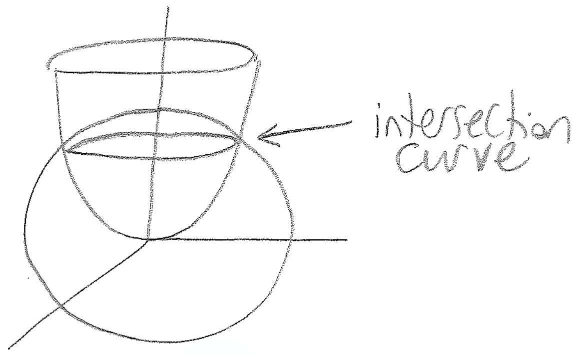
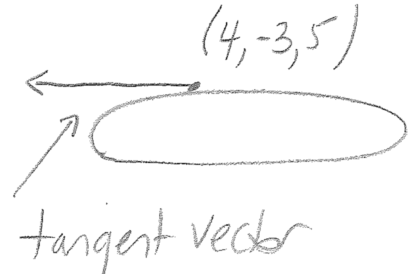
1





$\underbrace{x^2+y^2-5z}_{f} = 0$ is a level surface.

$\nabla f = [2x, 2y - 5]$

$= [8, -6, -5]$

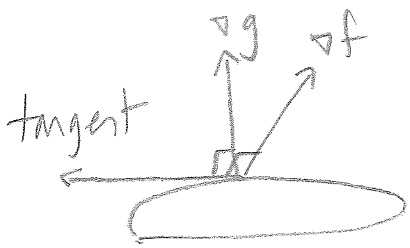
is normal to this surface, and therefore normal to the intersection.

$\underbrace{x^2+y^2+z^2}_{g} = 50$ is a level surface.

$\nabla g = [2x, 2y, 2z]$

$= [8, -6, 10]$

is normal to this surface, and therefore normal to the intersection.



tangent $= \nabla f \times \nabla g$

$= [-90, -120, 0]$

or any nonzero multiple of this

(2) Constraint: $\underbrace{x^2 + \frac{y^2}{4} + \frac{z^2}{16}}_{g} = 48$

$$\nabla f = \lambda \nabla g$$

$$[8yz, 8xz, 8xy] = \lambda\left[2x, \frac{y}{2}, \frac{z}{8}\right]$$

$$8yz = \lambda(2x) \Rightarrow \lambda = \frac{4yz}{x}$$

$$8xz = \lambda\left(\frac{y}{2}\right) \Rightarrow \lambda = \frac{16xz}{y}$$

$$8xy = \lambda\left(\frac{z}{8}\right) \Rightarrow \lambda = \frac{64xy}{z}$$

$$\lambda = \lambda = \lambda$$

$$\frac{4yz}{x} = \frac{16xz}{y} = \frac{64xy}{z}$$

$$\frac{4yz}{x} = \frac{16xz}{y}$$

$$\frac{4y}{x} = \frac{16x}{y}$$

$$4y^2 = 16x^2$$

$$y^2 = 4x^2$$

$$y = \pm 2x$$

but all variables are positive

$$\boxed{y = 2x}$$

$$\frac{4yz}{x} = \frac{64xy}{z}$$

$$\frac{4z}{x} = \frac{64x}{z}$$

$$4z^2 = 64x^2$$

$$z^2 = 16x^2$$

$$z = \pm 4x$$

but all variables are positive

$$\boxed{z = 4x}$$

$$\begin{aligned} y &= 2x \\ z &= 4x \end{aligned} \rightarrow x^2 + \frac{y^2}{4} + \frac{z^2}{16} = 48$$

$$x^2 + x^2 + x^2 = 48$$

$$3x^2 = 48$$

$$x^2 = 16$$

$$x = \pm 4$$

but all variables are positive

$$x = 4 \Rightarrow y = 8 \text{ and } z = 16$$

$$f(4,8,16) = 8(4)(8)(16) = 4096$$

The maximum value of f is 4096, achieved at $(x,y,z) = (4,8,16)$.

③ a) $f = -3\ln x + 12x + \frac{1}{y} + 9y$

$f_x = -\frac{3}{x} + 12$

$f_y = -\frac{1}{y^2} + 9$

$\left.\begin{array}{l} f_x \\ f_y \end{array}\right\}$ both 0 or undefined

Note: We are only interested in critical points with $x > 0$ and $y > 0$ in this problem.

$-\frac{3}{x} + 12 = 0$

$-3 + 12x = 0$

$-3 = -12x$

$x = \frac{1}{4}$

AND

$-\frac{1}{y^2} + 9 = 0$

$-1 + 9y^2 = 0$

$9y^2 = 1$

$y^2 = \frac{1}{9}$

$y = \pm\frac{1}{3}$

$y = \frac{1}{3}$

$(x, y) = \left(\frac{1}{4}, \frac{1}{3}\right)$

b)

Point	$f_{xx} = \frac{3}{x^2}$	$f_{xy} = 0$	$f_{yy} = \frac{2}{y^3}$	Δ	Conclude
$\left(\frac{1}{4}, \frac{1}{3}\right)$	48	0	54	>0	Local Minimum

(4) $$\int_0^{\frac{\pi}{2}}\int_0^3 \left(r\sqrt{9-r^2} + \sin\theta\right) dr\, d\theta$$

Let $I = \int r\sqrt{9-r^2}\, dr$

Let $u = 9-r^2$

$du = -2r\,dr$

$-\frac{1}{2}du = r\,dr$

$I = -\frac{1}{2}\int \sqrt{u}\,du$

$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$= -\frac{1}{3}(9-r^2)^{3/2} + C$

$$= \int_0^{\frac{\pi}{2}} \left[-\frac{1}{3}(9-r^2)^{3/2} + r\sin\theta\right]_{r=0}^{r=3} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left[3\sin\theta + \frac{1}{3}(9)^{3/2}\right] d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left[3\sin\theta + 9\right] d\theta$$

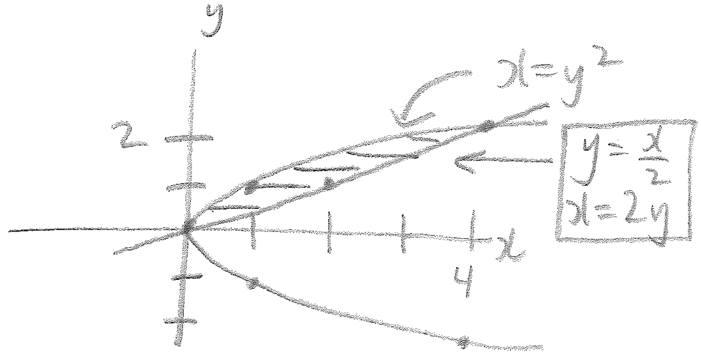
$$= \left[-3\cos\theta + 9\theta\right]_0^{\pi/2}$$

$$= 9\left(\frac{\pi}{2}\right) + 3$$

$$= \frac{9\pi}{2} + 3$$

or $\frac{9\pi+6}{2}$

(5)



y	$x = y^2$	
0	0	$(x,y) = (0,0)$
± 1	1	$(x,y) = (1, \pm 1)$
± 2	4	$(x,y) = (4, \pm 2)$

R: $\quad y^2 \le x \le 2y$

$\quad\quad 0 \le y \le 2$

$dA = dx\,dy$ (horizontal slices)

For every point in R,

$$\underbrace{x+3}_{z_{bottom}} < \underbrace{10}_{z_{top}}$$

$$V = \iint_R (z_{top} - z_{bottom})\, dA$$

$$= \int_0^2 \int_{y^2}^{2y} \left[10 - (x+3) \right] dx\, dy$$

or $$\int_0^2 \int_{y^2}^{2y} (7 - x)\, dx\, dy$$