1 a) Paraboloid

$z = x^2 + y^2$

$z = -x^2 - y^2$

$z = 1 - x^2 - y^2$

b) Cone

$z = \sqrt{x^2 + y^2}$

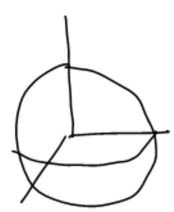
$z = -\sqrt{x^2 + y^2}$

$z = 1 - \sqrt{x^2 + y^2}$

c) Sphere

$z^2 = 1 - x^2 - y^2$

$x^2 + y^2 + z^2 = 1$



(2) $$f = e^{2x}\cos(y-3z) + z^2\sin(3x-2y)$$

$$f_x = 2e^{2x}\cos(y-3z) + 3z^2\cos(3x-2y)$$

$$f_y = -e^{2x}\sin(y-3z) - 2z^2\cos(3x-2y)$$

$$f_z = 3e^{2x}\sin(y-3z) + 2z\sin(3x-2y)$$

(3) $z_x = 6x + 5y - 3$

$z_y = 5x + 4y - 2$

Set $z_x = 0$ and $z_y = 0$:

$$6x + 5y - 3 = 0 \Rightarrow 6x + 5y = 3$$
$$5x + 4y - 2 = 0 \Rightarrow 5x + 4y = 2$$

Solve the system using any method, say Cramer's Rule.

$$x = \frac{\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}}{\begin{vmatrix} 6 & 5 \\ 5 & 4 \end{vmatrix}} = \frac{2}{-1} = -2$$

$$y = \frac{\begin{vmatrix} 6 & 3 \\ 5 & 2 \end{vmatrix}}{\begin{vmatrix} 6 & 5 \\ 5 & 4 \end{vmatrix}} = \frac{-3}{-1} = 3$$

$(x, y) = (-2, 3) \Rightarrow z = 3x^2 + 5xy + 2y^2 - 3x - 2y + 7$

$= 12 - 30 + 18 + 6 - 6 + 7$

$= 7$

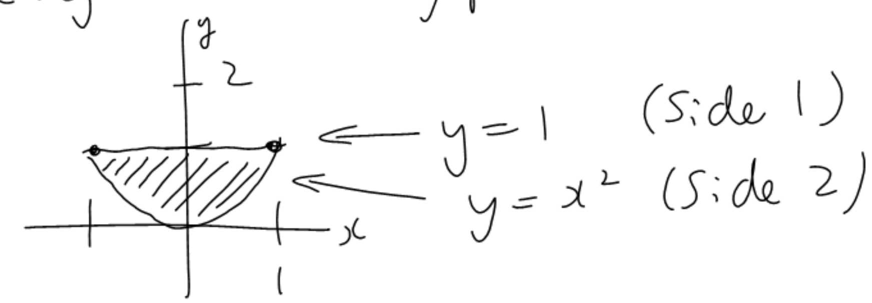
$$\boxed{(x, y, z) = (-2, 3, 7)}$$

④ 1) Interior Critical Points

$$\left.\begin{aligned} z_x &= 4x-2 \\ z_y &= 3 \end{aligned}\right\} \text{both 0 or undefined}$$

No interior critical points.

The region of the xy-plane looks like:



2) Corners

$(\pm 1, 1)$

3) Critical Points on side 1: $y=1$

$z = 2x^2 - 2x + 3y$

becomes $z = 2x^2 - 2x + 3$

$z' = 4x - 2$

set $z' = 0$: $4x - 2 = 0$

$x = \frac{1}{2}$

$y = 1$

Continued →

4) Critical Points on side 2: $y = x^2$

$$z = 2x^2 - 2x + 3y$$

becomes $z = 2x^2 - 2x + 3x^2$

$$z = 5x^2 - 2x$$

$$z' = 10x - 2$$

Set $z' = 0$: $\quad 10x - 2 = 0$

$$x = \frac{1}{5}$$

$$y = x^2 = \frac{1}{25}$$

Points	$z = 2x^2 - 2x + 3y$
$(1,1)$	3
$(-1,1)$	7
$(\frac{1}{2},1)$	$\frac{5}{2}$
$(\frac{1}{5},\frac{1}{25})$	$\frac{-1}{5}$

$\boxed{\text{The absolute minimum is } z = \frac{-1}{5}, \text{ achieved at } (x,y) = (\frac{1}{5}, \frac{1}{25}).}$