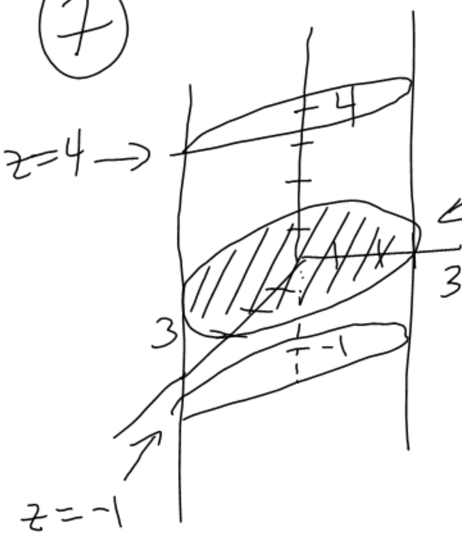
(7)




R } Solid cylinder Q

slice in z-direction:

$$-1 \le z \le 4$$

Project on xy-plane:

$$x^2+y^2=9$$

$$r^2 = 9$$

$$r = \pm 3$$

$$r = 3$$

R: $0 \le r \le 3$

$\quad 0 \le \theta \le 2\pi$

$$\vec{F} = [x^3, y^3, z^3]$$

$$\operatorname{div}\vec{F} = 3x^2+3y^2+3z^2$$

$$= 3(x^2+y^2+z^2)$$

$$= 3(r^2+z^2)$$

$\longrightarrow$

⑦ Cont'd

$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iiint_Q \operatorname{div}\vec{F}\,dV$$

$$= \int_0^{2\pi}\int_0^3\int_{-1}^4 3(r^2+z^2)\,\underline{\underline{r}}\,dz\,dr\,d\theta$$

$$= 3\int_0^{2\pi}\int_0^3\int_{-1}^4 (r^3+rz^2)\,dz\,dr\,d\theta$$

$$= 3\int_0^{2\pi}\int_0^3 \left[r^3z+\frac{rz^3}{3}\right]_{-1}^4 dr\,d\theta$$

$$= 3\int_0^{2\pi}\int_0^3 [\underbrace{4r^3+\frac{64}{3}r+r^3+\frac{r}{3}}_{5r^3+\frac{65}{3}r}]\,dr\,d\theta$$

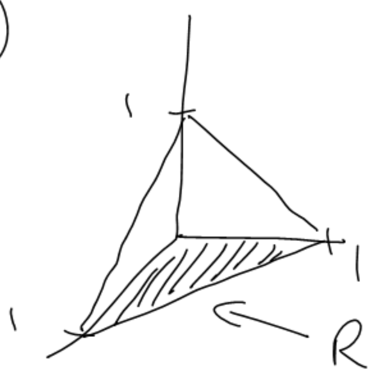
$$= 3\int_0^{2\pi}\left[\frac{5r^4}{4}+\frac{65r^2}{6}\right]_0^3 d\theta$$

$$= 3\left(\frac{795}{4}\right)\int_0^{2\pi} d\theta$$

$$= 3\left(\frac{795}{4}\right)(2\pi)$$

$$= \frac{2385\pi}{2}$$

9




$x+y+z=1$

$z=1-x-y$

Q is the solid tetrahedron.

Slice in the z-direction:

$0 \le z \le 1-x-y$

Project on xy-plane:


R: $0 \le y \le 1-x$

$0 \le x \le 1$

$\vec{F} = [x^2+e^{-yz}, y+\sin xz, \cos xy]$

$\operatorname{div} \vec{F} = 2x+1$

$\iint_S \vec{F}\cdot\vec{n}\,dS = \iiint_Q \operatorname{div}\vec{F}\,dV$

$\longrightarrow$

9 Cont'd

$$= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} (2x+1)\, dz\, dy\, dx$$

$$= \int_0^1 \int_0^{1-x} (2x+1)\, z \Big|_{z=0}^{z=1-x-y} dy\, dx$$

$$= \int_0^1 \int_0^{1-x} \underbrace{(2x+1)(1-x-y)} dy\, dx$$

$$(2x+1)[(1-x)-y]$$
$$= (2x+1)(1-x) - (2x+1)y$$

$$= \int_0^1 \left[(2x+1)(1-x)y - (2x+1)\frac{y^2}{2}\right]_{y=0}^{y=1-x} dx$$

$$= \int_0^1 \underbrace{\left[(2x+1)(1-x)^2 - \frac{1}{2}(2x+1)(1-x)^2\right]} dx$$

$$\frac{1}{2}(2x+1)(1-x)^2$$
$$= \frac{1}{2}(2x+1)(1-2x+x^2)$$
$$= \frac{1}{2}(2x - 4x^2 + 2x^3 + 1 - 2x + x^2)$$
$$= \frac{1}{2}(1 - 3x^2 + 2x^3)$$

$\longrightarrow$

9 Cont'd

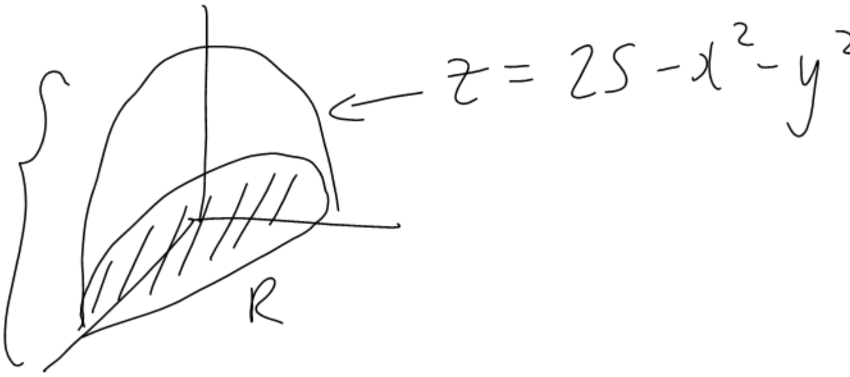
$$= \frac{1}{2} \int_0^1 (1 - 3x^2 + 2x^3)\, dx$$

$$= \frac{1}{2} \left[ x - x^3 + \frac{x^4}{2} \right]_0^1$$

$$= \frac{1}{2} \left[ 1 - 1 + \frac{1}{2} \right]$$

$$= \frac{1}{4}$$

11

Q 


Slice in z-direction:

$$0 \le z \le 25 - x^2 - y^2$$

$$0 \le z \le 25 - r^2$$

Project on xy-plane:

$$z = z$$

$$0 = 25 - x^2 - y^2$$

$$x^2 + y^2 = 25$$

$$r^2 = 25$$

$$r = \pm 5$$

$$r = 5$$

R: $0 \le r \le 5$

$0 \le \theta \le 2\pi$

$$\vec{F} = (x^2+y^2+z^2)[x, y, z]$$

$$= [x^3 + x(y^2+z^2),\ y^3 + y(x^2+z^2),\ z^3 + z(x^2+y^2)]$$

$\rightarrow$

(11) Cont'd

$$\operatorname{div} \vec{F} = 3x^2 + (y^2+z^2) + 3y^2 + (x^2+z^2) + 3z^2 + (x^2+y^2)$$
$$= 5x^2 + 5y^2 + 5z^2$$
$$= 5(x^2+y^2+z^2)$$
$$= 5(r^2+z^2)$$

$$\iint_S \vec{F}\cdot\vec{n}\, dS = \iiint_Q \operatorname{div}\vec{F}\, dV$$
$$= 5\int_0^{2\pi}\int_0^5\int_0^{25-r^2} (r^2+z^2)\, \underline{\underline{r}}\, dz\, dr\, d\theta$$
$$= 5\int_0^{2\pi}\int_0^5\int_0^{25-r^2} (r^3+rz^2)\, dz\, dr\, d\theta$$
$$= 5\int_0^{2\pi}\int_0^5 \left[r^3 z + \frac{rz^3}{3}\right]_{z=0}^{z=25-r^2} dr\, d\theta$$
$$= 5\int_0^{2\pi}\int_0^5 \left[\underbrace{r^3(25-r^2)}_{25r^3-r^5} + \underbrace{\frac{r}{3}(25-r^2)^3}_{\text{Substitution}}\right] dr\, d\theta$$

$\longrightarrow$

(11) Cont'd

Substitution

$u = 25 - r^2$

$du = -2r\,dr$

$-\frac{1}{2}du = r\,dr$

$\int \frac{r}{3}(25-r^2)^3 dr = -\frac{1}{6}\int u^3 du$

$= -\frac{1}{24}u^4 + C$

$= -\frac{1}{24}(25-r^2)^4 + C$

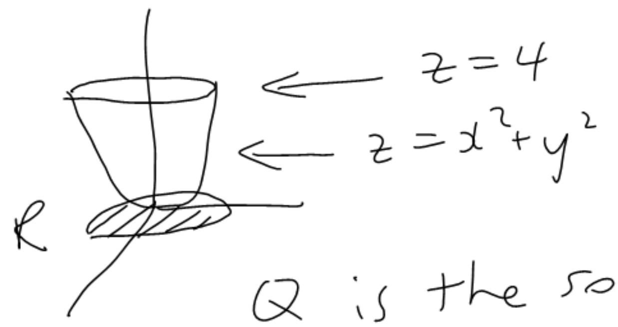
$$= 5\int_0^{2\pi}\left[\frac{25r^4}{4} - \frac{r^6}{6} - \frac{(25-r^2)^4}{24}\right]_0^5 d\theta$$

$$= 5\left[\frac{25(5^4)}{4} - \frac{5^6}{6} + \frac{25^4}{24}\right]\int_0^{2\pi} d\theta$$

$$= 5\,\frac{140625}{8}(2\pi)$$

$$= \frac{703125\pi}{4}$$

(13)




Q is the solid region.

Slice in the z-direction:

$$x^2+y^2 \le z \le 4$$
$$r^2 \le z \le 4$$

Project on the xy-plane:

$$z=z$$
$$x^2+y^2=4$$
$$r^2=4$$
$$r=\pm 2$$
$$r=2$$

R: $0 \le r \le 2$
$0 \le \theta \le 2\pi$

$$\vec{F} = [x, y, 3]$$
$$\operatorname{div}\vec{F} = 1+1+0$$
$$= 2$$

$\longrightarrow$

(13) Cont'd

$$\iint_S \vec{F}\cdot\vec{n}\, dS = \iiint_Q \operatorname{div}\vec{F}\, dV$$

$$= \int_0^{2\pi}\int_0^2\int_{r^2}^4 2r\,dz\,dr\,d\theta$$

$$= 2\int_0^{2\pi}\int_0^2 rz\Big|_{z=r^2}^{z=4} dr\,d\theta$$

$$= 2\int_0^{2\pi}\int_0^2 (4r-r^3)\,dr\,d\theta$$

$$= 2\int_0^{2\pi}\left[2r^2-\frac{r^4}{4}\right]_0^2 d\theta$$

$$= 2\,[8-4]\int_0^{2\pi} d\theta$$

$$= 8(2\pi)$$

$$= 16\pi$$