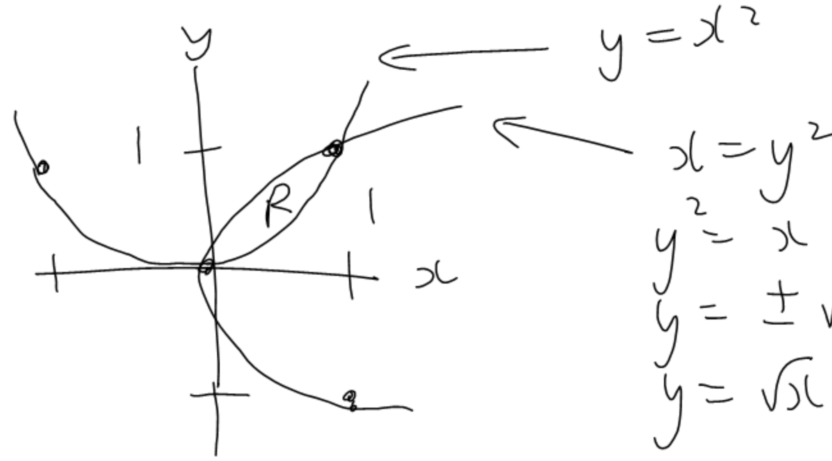
(5) $P = -y^2 + e^{(e^x)}$, $Q = \arctan y$



$y^2 = x$

$y = \pm\sqrt{x}$

$y = \sqrt{x}$ is the top half

$$\boxed{R: \quad x^2 \le y \le \sqrt{x} \\ 0 \le x \le 1}$$

$$\frac{\partial Q}{\partial x} = 0 \quad , \quad \frac{\partial P}{\partial y} = -2y$$

$$\oint_C (P\,dx + Q\,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} 2y \; dy \, dx$$

$$= \int_0^1 y^2 \Big|_{y=x^2}^{y=\sqrt{x}} dx$$

$\longrightarrow$

(5) Cont'd

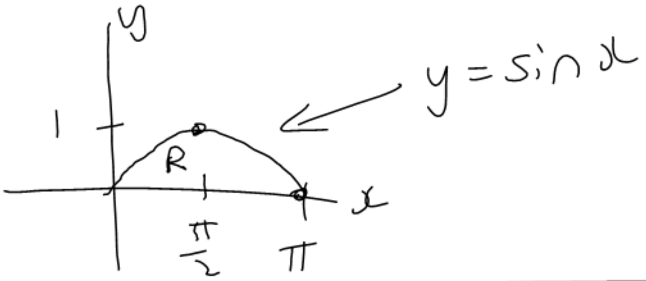
$$= \int_0^1 (x - x^4)\, dx$$

$$= \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1$$

$$= \frac{1}{2} - \frac{1}{5}$$

$$= \frac{3}{10}$$

7. $P = x - y, \quad Q = y$



$$\boxed{R: \quad 0 \le y \le \sin x, \quad 0 \le x \le \pi}$$

$$\frac{\partial Q}{\partial x} = 0, \quad \frac{\partial P}{\partial y} = -1$$

$$\oint_C (P\,dx + Q\,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \int_0^{\pi} \int_0^{\sin x} 1 \, dy \, dx$$

$$= \int_0^{\pi} y \Big|_{y=0}^{y=\sin x} dx$$

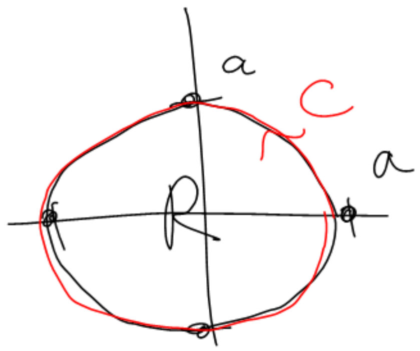
$$= \int_0^{\pi} \sin x \, dx$$

$$= \left[-\cos x\right]_0^{\pi}$$

$$= 1 - (-1)$$

$$= 2$$

(13)



$x = a\cos t$

$y = a\sin t$

$0 \le t \le 2\pi$

$dy = a\cos t \, dt$

$$A = \oint_C x \, dy$$

$$= \int_0^{2\pi} a\cos t \, (a\cos t \, dt)$$

$$= a^2 \int_0^{2\pi} \underbrace{\cos^2 t}_{\boxed{\frac{1+\cos 2t}{2}}} \, dt$$

$$= \frac{a^2}{2} \int_0^{2\pi} (1+\cos 2t) \, dt$$

$$= \frac{a^2}{2} \left[t + \frac{\sin 2t}{2} \right]_0^{2\pi}$$

$$= \frac{a^2}{2} [2\pi - 0]$$

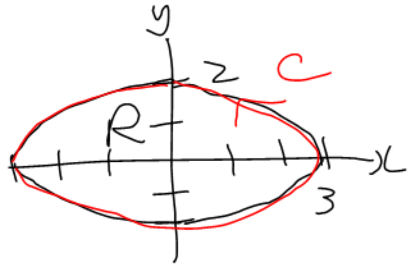
$$= \pi a^2$$ (as expected)

(17) $\vec{F} = [-2y, 3x]$

$P = -2y, \quad Q = 3x$

$\frac{\partial P}{\partial y} = -2, \quad \frac{\partial Q}{\partial x} = 3$

$C: \frac{x^2}{9} + \frac{y^2}{4} = 1$



$$W = \oint_C \vec{F} \cdot \vec{T}\, ds$$

$$= \oint_C (P\,dx + Q\,dy)$$

this is the calculation formula for Work

$$= \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

by Green's Theorem

$$= \iint_R 5\, dA$$

$$= 5 \boxed{\iint_R dA}$$

← area of R

Recall area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab

$$= 5\pi(3)(2)$$

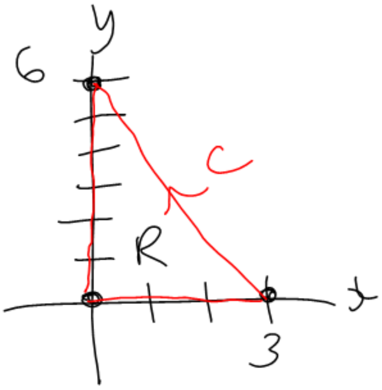
$$= 30\pi$$

(23) $\text{div}\,\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$

$$= \frac{\partial}{\partial x}\left(3x + \sqrt{1+y^2}\right) + \frac{\partial}{\partial y}\left(2y - \sqrt[3]{1+x^4}\right)$$

$$= 3 + 2$$

$$= 5$$



By 2D Divergence Theorem,

$$\Phi = \iint_R (\text{div}\,\vec{F})\,dA$$

$$= \iint_R 5\,dA$$

$$= 5 \underbrace{\boxed{\iint_R dA}}$$ ← area of R

$$= 5\left(\tfrac{1}{2}bh\right)$$

$$= 5\left(\tfrac{1}{2}\right)(3)(6)$$

$$= 45$$