1. $\vec{F} = [2x+3y,\ 3x+2y]$

$$\frac{\partial Q}{\partial x} = 3 = \frac{\partial P}{\partial y}$$

Yes, $\vec{F}$ is conservative.

$f = \int (2x+3y)\,dx$ AND $f = \int (3x+2y)\,dy$

$= x^2 + 3xy + g(y)$ $\qquad = 3xy + y^2 + h(x)$

$$\Rightarrow f = x^2 + 3xy + y^2 + C$$

(3) $\vec{F} = [3x^2+2y^2,\ 4xy+6y^2]$

$$\frac{\partial Q}{\partial x} = 4y = \frac{\partial P}{\partial y}$$

Yes, $\vec{F}$ is conservative.

$f = \int (3x^2+2y^2)\,dx$ AND $f = \int (4xy+6y^2)\,dy$

$= x^3 + 2xy^2 + g(y)$ $\quad = 2xy^2 + 2y^3 + h(x)$

$$\Rightarrow \quad f = x^3 + 2xy^2 + 2y^3 + C$$

⑤ $\vec{F} = [2y + \sin 2x,\ 3x + \cos 3y]$

$$\frac{\partial Q}{\partial x} = 3 \qquad \frac{\partial P}{\partial y} = 2$$

$$\frac{\partial Q}{\partial x} \neq \frac{\partial P}{\partial y}$$

No, $\vec{F}$ is not conservative.

(11) $\vec{F} = [x\cos y + \sin y,\ y\cos x + \sin x]$

$$\frac{\partial Q}{\partial x} = -y\sin x + \cos x$$

$$\frac{\partial P}{\partial y} = -x\sin y + \cos y$$

$$\frac{\partial Q}{\partial x} \neq \frac{\partial P}{\partial y}$$

No, $\vec{F}$ is not conservative.

(23) $\displaystyle\int_{(0,0)}^{(1,-1)} \left(2xe^y\,dx + x^2e^y\,dy\right)$

$$= \int_{(0,0)}^{(1,-1)} \underbrace{[2xe^y,\ x^2e^y]}_{\vec{F}} \cdot \underbrace{[dx,\ dy]}_{d\vec{r}}$$

$P = 2xe^y$
$Q = x^2e^y$

$$\frac{\partial Q}{\partial x} = 2xe^y = \frac{\partial P}{\partial y}$$

$\Rightarrow$ $\vec{F}$ is conservative
$\Rightarrow$ The line integral is path-independent.

Find a potential :

$f = \int 2xe^y\,dx$ AND $f = \int x^2e^y\,dy$
$= x^2e^y + g(y)$ $\qquad$ $= x^2e^y + h(x)$

$\Rightarrow$ $f = x^2e^y + C$ $\qquad$ (can omit $+C$)

Integral $= f(B) - f(A)$, where A and B are the endpoints

$$= \left[x^2e^y + C\right]_{(0,0)}^{(1,-1)}$$

$\rightarrow$

23 Cont'd

$$= e^{-1} + C - C$$

$$= e^{-1}$$

(25) $$\int_{(\frac{\pi}{2},\frac{\pi}{2})}^{(\pi,\pi)} \left[(\sin y + y\cos x)\,dx + (\sin x + x\cos y)\,dy\right]$$

$$= \int_{(\frac{\pi}{2},\frac{\pi}{2})}^{(\pi,\pi)} \underbrace{[\sin y + y\cos x,\ \sin x + x\cos y]}_{\vec{F}} \cdot \underbrace{[dx, dy]}_{d\vec{r}}$$

$$P = \sin y + y\cos x$$
$$Q = \sin x + x\cos y$$

$$\frac{\partial Q}{\partial x} = \cos x + \cos y = \frac{\partial P}{\partial y}$$

$\Rightarrow$ $\vec{F}$ is conservative.

$\Rightarrow$ The line integral is path-independent.

Find a potential:

$f = \int (\sin y + y\cos x)\,dx$ AND $f = \int (\sin x + x\cos y)\,dy$

$= x\sin y + y\sin x + g(y)$ $\qquad = y\sin x + x\sin y + h(x)$

$\Rightarrow f = x\sin y + y\sin x + C$ (can omit $+C$)

$\longrightarrow$

(25) Cont'd

Integral $= f(B) - f(A)$, where
A and B are the endpoints

$$= \Big[x\sin y + y\sin x + C \Big]_{(\frac{\pi}{2}, \frac{\pi}{2})}^{(\pi, \pi)}$$

$$= 0 + C - \left[\frac{\pi}{2} + \frac{\pi}{2} + C \right]$$

$$= -\pi$$

(27) It's implied that $\vec{F}$ is conservative.
(You could check that $\operatorname{curl}\vec{F} = \vec{0}$.)

$f = \int yz\,dx$ AND $f = \int xz\,dy$ AND

$f = \int xy\,dz$

$\Rightarrow$ $f = xyz + g(y,z)$ AND
$f = xyz + h(x,z)$ AND
$f = xyz + K(x,y)$

$\Rightarrow$ $f = xyz + C$

(29) It's implied that $\vec{F}$ is conservative.
(You could check that curl $\vec{F} = \vec{0}$.)

$$f = \int (y\cos z - yze^{x})\,dx \quad \text{AND}$$
$$f = \int (x\cos z - ze^{x})\,dy \quad \text{AND}$$
$$f = \int -(xy\sin z + ye^{x})\,dz = \int (-xy\sin z - ye^{x})\,dz$$

$$\Rightarrow \quad f = xy\cos z - yze^{x} + g(y,z) \quad \text{AND}$$
$$f = xy\cos z - yze^{x} + h(x,z) \quad \text{AND}$$
$$f = xy\cos z - yze^{x} + k(x,y)$$

$$\Rightarrow f = xy\cos z - yze^{x} + C$$

(33) To show that the integral is <u>not</u> path-independent we must show that $\vec{F}$ is <u>not</u> conservative.

$$\int_C (2xy\,dx + x^2 dy + y^2 dz)$$

$$= \int_C \underbrace{[2xy, x^2, y^2]}_{\vec{F}} \cdot \underbrace{[dx, dy, dz]}_{d\vec{r}}$$

$$\text{curl}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & x^2 & y^2 \end{vmatrix}$$

$$= \vec{i}\,(2y) - \vec{j}\,(0) + \vec{k}\,(2x - 2x)$$

$$= [2y, 0, 0]$$

$$\neq \vec{0}$$

$\vec{F}$ is not conservative

$\Rightarrow$ the integral is path-dependent (not path-independent).