1

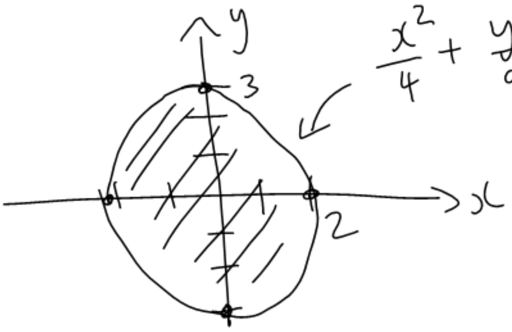
$z = x + 3y$

$z_x = 1 \qquad z_y = 3$

$\sqrt{1 + (z_x)^2 + (z_y)^2} = \sqrt{1 + 1 + 9}$

$= \sqrt{11}$

R:



$\frac{x^2}{4} + \frac{y^2}{9} = 1$

$\boxed{\text{Recall: Area of the ellipse } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } \pi ab.}$

$$SA = \iint_R \sqrt{1 + (z_x)^2 + (z_y)^2}\, dA$$

$$= \iint_R \sqrt{11}\, dA$$

$$= \sqrt{11} \underbrace{\iint_R dA}_{\text{area of ellipse}}$$

$$= \sqrt{11}\, \pi (2)(3)$$

$$= 6\sqrt{11}\, \pi$$

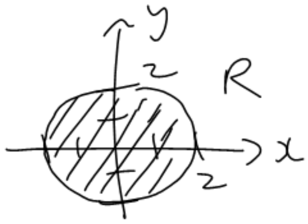
(3)

$$z = 9 - x^2 - y^2$$

$$z_x = -2x \qquad z_y = -2y$$

$$\sqrt{1+(z_x)^2+(z_y)^2} = \sqrt{1+4x^2+4y^2}$$
$$= \sqrt{1+4(x^2+y^2)}$$
$$= \sqrt{1+4r^2}$$

R:
$$z = z$$
$$9-x^2-y^2 = 5$$
$$4 = x^2+y^2$$
$$r^2 = 4$$
$$r = \pm 2$$
$$r = 2$$



$$SA = \iint_R \sqrt{1+(z_x)^2+(z_y)^2}\, dA$$
$$= \int_0^{2\pi}\int_0^{2} \sqrt{1+4r^2}\; r\,dr\,d\theta$$

Sub $u = 1+4r^2$
$du = 8r\,dr$
$\frac{du}{8} = r\,dr$
$r=0 \Rightarrow u=1$
$r=2 \Rightarrow u=17$

$$= \frac{1}{8}\int_0^{2\pi}\int_1^{17} \sqrt{u}\, du\, d\theta$$

$\rightarrow$

③ Cont'd

$$= \frac{1}{8}\cdot\frac{2}{3}\int_0^{2\pi} u^{3/2}\Big|_1^{17} \, d\theta$$

$$= \frac{1}{8}\cdot\frac{2}{3}\left[17^{3/2}-1\right]\int_0^{2\pi} d\theta$$

$$= \frac{1}{12}\left[17^{3/2}-1\right](2\pi)$$

$$= \frac{\pi}{6}\left(17^{3/2}-1\right)$$

(7) $2x + 3y + z = 6$

$$z = 6 - 2x - 3y$$

$$z_x = -2 \qquad z_y = -3$$

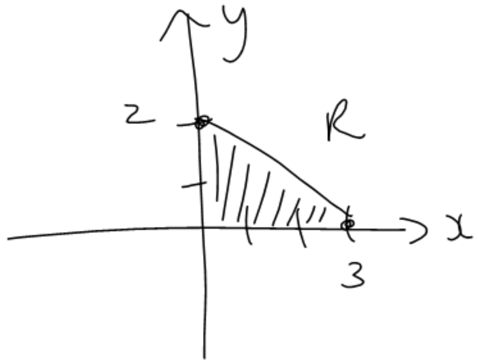
$$\sqrt{1 + (z_x)^2 + (z_y)^2} = \sqrt{1+4+9}$$
$$= \sqrt{14}$$

R: $z = z$

$6 - 2x - 3y = 0$

$6 = 2x + 3y$

$2x + 3y = 6$



$$SA = \iint_R \sqrt{1 + (z_x)^2 + (z_y)^2}\, dA$$

$$= \iint_R \sqrt{14}\, dA$$

$$= \sqrt{14} \underbrace{\iint_R dA}_{\text{area of } R}$$

$$= \sqrt{14}\left(\tfrac{1}{2} \cdot \text{base} \cdot \text{height}\right)$$

$$= \sqrt{14}\left(\tfrac{1}{2} \cdot 3 \cdot 2\right)$$

$$= 3\sqrt{14}$$

(9) $z = xy$

$z_x = y \quad z_y = x$

$\sqrt{1+(z_x)^2+(z_y)^2} = \sqrt{1+y^2+x^2}$

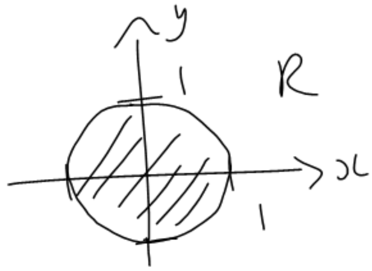
$= \sqrt{1+r^2}$

$R: \quad x^2+y^2 = 1$

$r^2 = 1$

$r = \pm 1$

$r = 1$



$$SA = \iint_R \sqrt{1+(z_x)^2+(z_y)^2}\, dA$$

$$= \int_0^{2\pi}\int_0^1 \sqrt{1+r^2}\, r\,dr\,d\theta$$

$$\boxed{\begin{aligned} &\text{Sub } u = 1+r^2 \\ &du = 2r\,dr \\ &\frac{du}{2} = r\,dr \\ &r=0 \Rightarrow u=1 \\ &r=1 \Rightarrow u=2 \end{aligned}}$$

$$= \frac{1}{2}\int_0^{2\pi}\int_1^2 \sqrt{u}\, du\, d\theta$$

$$= \frac{1}{2}\cdot\frac{2}{3}\int_0^{2\pi} u^{3/2}\Big|_1^2 \, d\theta$$

9 Cont'd

$$= \frac{1}{3} \int_0^{2\pi} (2\sqrt{2} - 1) \, d\theta$$

$$= \frac{1}{3} (2\sqrt{2} - 1) \int_0^{2\pi} d\theta$$

$$= \frac{2\pi}{3} (2\sqrt{2} - 1)$$

(13) $\vec{r} = [u\cos\theta,\ u\sin\theta,\ u]$

where $0 \le u \le 7,\ 0 \le \theta \le 2\pi$

$\vec{r}_u = [\cos\theta,\ \sin\theta,\ 1]$

$\vec{r}_\theta = [-u\sin\theta,\ u\cos\theta,\ 0]$

$$\vec{r}_u \times \vec{r}_\theta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\theta & \sin\theta & 1 \\ -u\sin\theta & u\cos\theta & 0 \end{vmatrix}$$

$$= \vec{i}(-u\cos\theta) - \vec{j}(u\sin\theta) + \vec{k}(u\cos^2\theta + u\sin^2\theta)$$

$$= [-u\cos\theta,\ -u\sin\theta,\ u]$$

$$\|\vec{r}_u \times \vec{r}_\theta\| = \sqrt{u^2\cos^2\theta + u^2\sin^2\theta + u^2}$$

$$= \sqrt{u^2(\cos^2\theta + \sin^2\theta + 1)}$$

$$= \sqrt{u^2(1+1)}$$

$= \sqrt{2}\,u$ (We were given $u \geq 0$).

$$SA = \iint_R \|\vec{r}_u \times \vec{r}_\theta\|\, du\, d\theta$$

$$= \int_0^{2\pi}\int_0^{7} \sqrt{2}\, u\, du\, d\theta$$

(13) Cont'd

$$= \sqrt{2} \int_0^{2\pi} \frac{u^2}{2} \Big|_0^7 \, d\theta$$

$$= \sqrt{2} \, \frac{49}{2} \int_0^{2\pi} d\theta$$

$$= \sqrt{2} \, \frac{49}{2} \, (2\pi)$$

$$= 49\sqrt{2} \, \pi$$