(3) T :
$$-2 \le z \le 6$$
$$0 \le y \le 2$$
$$-1 \le x \le 3$$

$$\iiint_T f\,dV = \int_{-1}^{3}\int_{0}^{2}\int_{-2}^{6} xyz\,dz\,dy\,dx$$

$$= \int_{-1}^{3}\int_{0}^{2} \frac{xyz^2}{2}\Big|_{z=-2}^{z=6} dy\,dx$$

$$= \int_{-1}^{3}\int_{0}^{2} [18xy - 2xy]\,dy\,dx$$

$$= \int_{-1}^{3}\int_{0}^{2} 16xy\,dy\,dx$$

$$= \int_{-1}^{3} 8xy^2\Big|_{y=0}^{y=2} dx$$

$$= \int_{-1}^{3} 32x\,dx$$

$$= 16x^2\Big|_{-1}^{3}$$

$$= 144 - 16$$

$$= 128$$

(5)

$x+y+z=1$
or $z = 1-x-y$

Slice in the z-direction:

$$0 \le z \le 1-x-y$$

Project on the xy-plane:

$$z = z$$
$$1-x-y = 0$$
$$1 = x+y$$
$$y = 1-x$$

And $x=0$, $y=0$

R: $0 \le y \le 1-x$

$0 \le x \le 1$

$$\iiint_T f\,dV = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^2\,dz\,dy\,dx$$

$$= \int_0^1 \int_0^{1-x} x^2 z \Big|_{z=0}^{z=1-x-y} dy\,dx$$

$$= \int_0^1 \int_0^{1-x} x^2(1-x-y)\,dy\,dx$$

$$= \int_0^1 \int_0^{1-x} (x^2 - x^3 - x^2 y)\,dy\,dx$$

Continued →

⑤ Cont'd

$$= \int_0^1 \left[x^2 y - x^3 y - \frac{x^2 y^2}{2} \right]_{y=0}^{y=1-x} dx$$

$$= \int_0^1 \left[x^2(1-x) - x^3(1-x) - \frac{x^2}{2}(1-x)^2 \right] dx$$

$$= \int_0^1 \left[x^2 - x^3 - x^3 + x^4 - \frac{x^2}{2}(1-2x+x^2) \right] dx$$

$$= \int_0^1 \left[x^2 - 2x^3 + x^4 - \frac{x^2}{2} + x^3 - \frac{x^4}{2} \right] dx$$

$$= \int_0^1 \left[\frac{x^2}{2} - x^3 + \frac{x^4}{2} \right] dx$$

$$= \left[\frac{x^3}{6} - \frac{x^4}{4} + \frac{x^5}{10} \right]_0^1$$

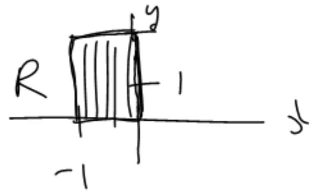
$$= \frac{1}{60}$$

⑦ Slice in the z-direction:

$0 \le z \le 1-x^2$

R is given : $0 \le y \le 2$

$-1 \le x \le 0$



$$\iiint_T f\,dV = \int_{-1}^{0}\int_{0}^{2}\int_{0}^{1-x^2} xyz \,dz\,dy\,dx$$

$$= \int_{-1}^{0}\int_{0}^{2} \frac{xyz^2}{2}\Bigg|_{z=0}^{z=1-x^2} dy\,dx$$

$$= \int_{-1}^{0}\int_{0}^{2} \frac{xy}{2}(1-x^2)^2 \,dy\,dx$$

$$= \int_{-1}^{0} \frac{xy^2}{4}(1-x^2)^2\Bigg|_{y=0}^{y=2} dx$$

$$= \int_{-1}^{0} x(1-x^2)^2\,dx$$

$$= \int_{-1}^{0} x(1-2x^2+x^4)\,dx$$

$$= \int_{-1}^{0} (x-2x^3+x^5)\,dx$$

$$= \left[\frac{x^2}{2} - \frac{x^4}{2} + \frac{x^6}{6}\right]_{-1}^{0}$$

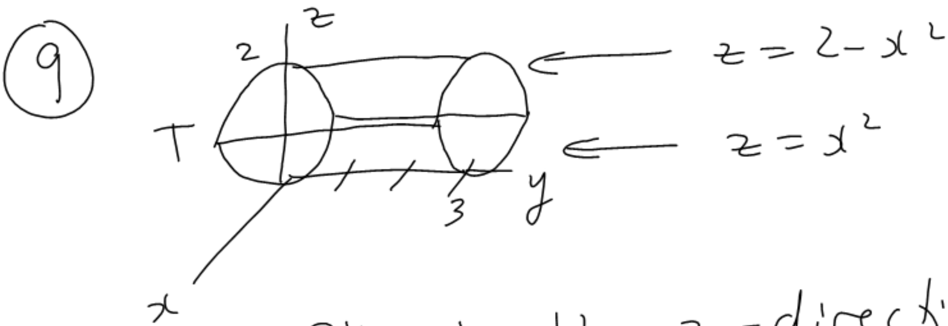
Continued →

⑦ Cont'd $= 0 - \left[\frac{1}{2} - \frac{1}{2} + \frac{1}{6} \right]$

$= \frac{-1}{6}$

Note: The value of the triple integral is negative because $f = xyz$ is negative at most points in the solid region T.

9



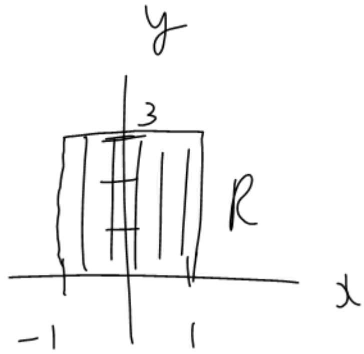
Slice in the z-direction:

$$x^2 \le z \le 2 - x^2$$

Project on xy-plane:

$$z = z$$
$$x^2 = 2 - x^2$$
$$2x^2 = 2$$
$$x^2 = 1$$
$$x = \pm 1$$

And $0 \le y \le 3$



R: $0 \le y \le 3$
$-1 \le x \le 1$

$$\iiint_T f\,dV = \int_{-1}^{1}\int_{0}^{3}\int_{x^2}^{2-x^2} (x+y)\,dz\,dy\,dx$$

$$= \int_{-1}^{1}\int_{0}^{3} (x+y)z \Big|_{z=x^2}^{z=2-x^2} dy\,dx$$

$$= \int_{-1}^{1}\int_{0}^{3} \left[(x+y)(2-x^2) - (x+y)x^2\right] dy\,dx$$

Continued →

⑨ Cont'd

$$= \int_{-1}^{1}\int_{0}^{3} (x+y)(2-2x^2)\ dy\, dx$$

$$= \int_{-1}^{1} \left(xy+\frac{y^2}{2}\right)(2-2x^2)\Big|_{y=0}^{y=3}\ dx$$

$$= \int_{-1}^{1} \left(3x+\frac{9}{2}\right)(2-2x^2)\ dx$$

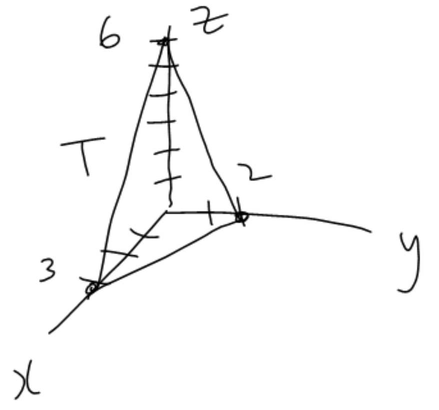
$$= \int_{-1}^{1} (6x-6x^3+9-9x^2)\ dx$$

$$= \left[3x^2-\frac{6x^4}{4}+9x-3x^3\right]_{-1}^{1}$$

$$= \left(3-\frac{3}{2}+9-3\right)-\left(3-\frac{3}{2}-9+3\right)$$

$$= 12$$

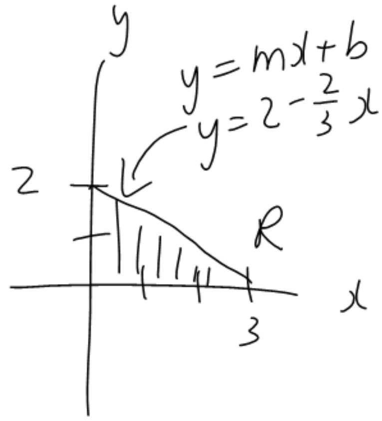
(11)



$$2x + 3y + z = 6$$
or $z = 6 - 2x - 3y$

Slice in the z-direction:
$$0 \le z \le 6 - 2x - 3y$$

Project on xy-plane:
$$z = z$$
$$6 - 2x - 3y = 0$$
$$-3y = -6 + 2x$$
$$y = 2 - \frac{2}{3}x$$
And $x = 0, y = 0$



R: $0 \le y \le 2 - \frac{2}{3}x$
$0 \le x \le 3$

$$V = \iiint_T dV$$
$$= \int_0^3 \int_0^{2-\frac{2}{3}x} \int_0^{6-2x-3y} dz\, dy\, dx$$
$$= \int_0^3 \int_0^{2-\frac{2}{3}x} z \Big|_{z=0}^{z=6-2x-3y} dy\, dx$$

Continued →

(11) Cont'd

$$= \int_0^3 \int_0^{2-\frac{2}{3}x} (6-2x-3y)\,dy\,dx$$

$$= \int_0^3 \left[(6-2x)y - \frac{3y^2}{2} \right]_{y=0}^{y=2-\frac{2}{3}x} dx$$

$$= \int_0^3 \left[(6-2x)\left(2-\frac{2}{3}x\right) - \frac{3}{2}\left(2-\frac{2}{3}x\right)^2 \right] dx$$

$$= \int_0^3 \left[12 - 4x - 4x + \frac{4}{3}x^2 - \frac{3}{2}\left(4 - \frac{8}{3}x + \frac{4}{9}x^2\right) \right] dx$$

$$= \int_0^3 \left[12 - 8x + \frac{4}{3}x^2 - 6 + 4x - \frac{2}{3}x^2 \right] dx$$

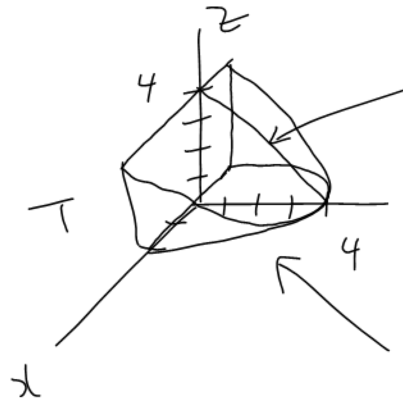
$$= \int_0^3 \left(6 - 4x + \frac{2}{3}x^2\right) dx$$

$$= \left[6x - 2x^2 + \frac{2}{9}x^3 \right]_0^3$$

$$= 18 - 18 + 6$$

$$= 6$$

(13)



$y+z=4$ or $z=4-y$
(top)

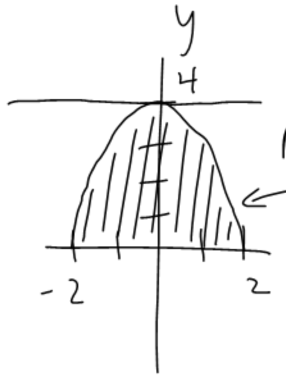
$y=4-x^2$
(side)

Slice in the z-direction:

$$0 \le z \le 4-y$$

Project on the xy-plane:

$$z = z$$
$$4-y = 0$$
$$4 = y$$
and $y=0$, $y=4-x^2$



$$R: \quad 0 \le y \le 4-x^2$$
$$-2 \le x \le 2$$

$$V = \iiint_T dV$$

$$= \int_{-2}^{2}\int_{0}^{4-x^2}\int_{0}^{4-y} dz\,dy\,dx$$

$$= \int_{-2}^{2}\int_{0}^{4-x^2} z \Big|_{z=0}^{z=4-y} dy\,dx$$

Continued →

(13) Cont'd

$$= \int_{-2}^{2} \int_{0}^{4-x^2} (4-y)\, dy\, dx$$

$$= \int_{-2}^{2} \left[4y - \frac{y^2}{2} \right]_{y=0}^{y=4-x^2} dx$$

$$= \int_{-2}^{2} \left[4(4-x^2) - \frac{1}{2}(4-x^2)^2 \right] dx$$

$$= \int_{-2}^{2} \left[16 - 4x^2 - \frac{1}{2}(16 - 8x^2 + x^4) \right] dx$$

$$= \int_{-2}^{2} \left[16 - 4x^2 - 8 + 4x^2 - \frac{x^4}{2} \right] dx$$

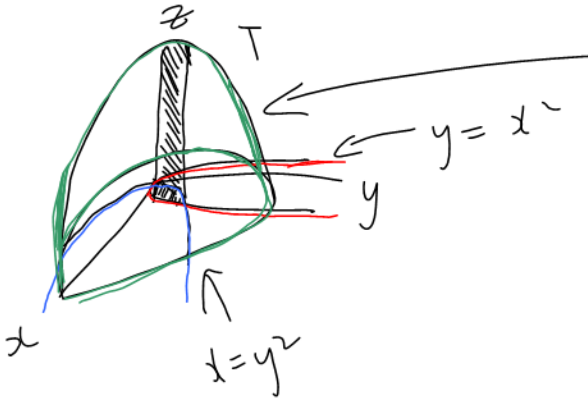
$$= \int_{-2}^{2} \left[8 - \frac{x^4}{2} \right] dx$$

$$= \left[8x - \frac{x^5}{10} \right]_{-2}^{2}$$

$$= \left(16 - \frac{32}{10}\right) - \left(-16 + \frac{32}{10}\right)$$

$$= \frac{128}{5}$$

(15)



$z = 10 - x^2 - y^2$

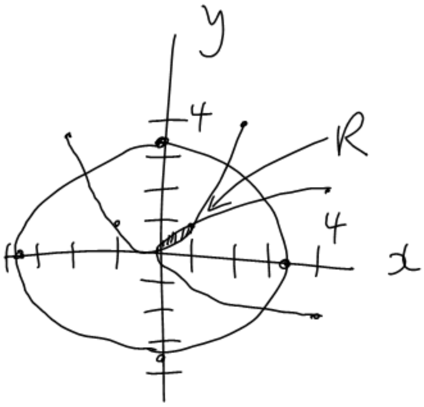
Slice in the z-direction:

$$0 \le z \le 10 - x^2 - y^2$$

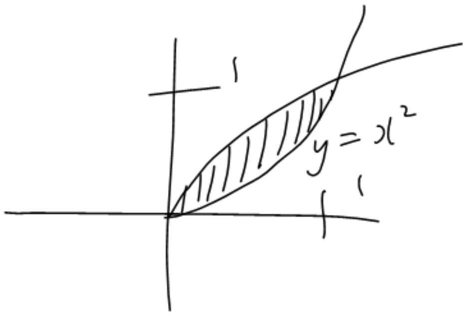
Projelt on xy-plane:

$$z = z$$
$$10 - x^2 - y^2 = 0$$
$$10 = x^2 + y^2$$
$$x^2 + y^2 = 10$$

and $y = x^2$, $x = y^2$



Zoomed-in view of R:



$$x = y^2$$
$$y^2 = x$$
$$y = \pm\sqrt{x}$$
$$y = \sqrt{x}$$

$$R: \quad x^2 \le y \le \sqrt{x}$$
$$0 \le x \le 1$$

Continued

15 Cont'd

$$V = \iiint_T dV$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{10-x^2-y^2} dz\, dy\, dx$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} z \Big|_{z=0}^{z=10-x^2-y^2} dy\, dx$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} (10-x^2-y^2)\, dy\, dx$$

$$= \int_0^1 \left[(10-x^2)y - \frac{y^3}{3} \right]_{y=x^2}^{y=\sqrt{x}} dx$$

$$= \int_0^1 \left[(10-x^2)\sqrt{x} - \frac{1}{3}x^{3/2} - (10-x^2)x^2 + \frac{x^6}{3} \right] dx$$

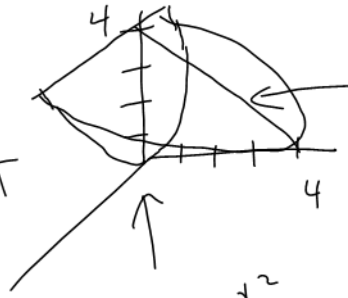
$$= \int_0^1 \left[10x^{1/2} - x^{5/2} - \frac{1}{3}x^{3/2} - 10x^2 + x^4 + \frac{x^6}{3} \right] dx$$

$$= \left[\frac{20}{3}x^{3/2} - \frac{2}{7}x^{7/2} - \frac{2}{15}x^{5/2} - \frac{10x^3}{3} + \frac{x^5}{5} + \frac{x^7}{21} \right]_0^1$$

$$= \frac{20}{3} - \frac{2}{7} - \frac{2}{15} - \frac{10}{3} + \frac{1}{5} + \frac{1}{21}$$

$$= \frac{332}{105}$$

(17)



$y+z=4$ or $z=4-y$
(top)

$z=x^2$
(side and bottom)

Slice in the z-direction:

$$x^2 \le z \le 4-y$$

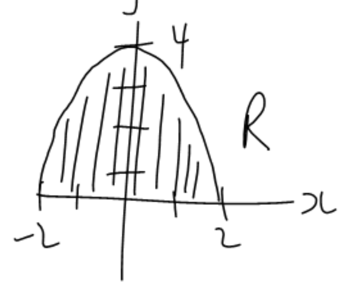
Project on the xy-plane:

$z=z$

$x^2 = 4-y$

$y = 4-x^2$

and $y=0$



R: $0 \le y \le 4-x^2$

$-2 \le x \le 2$

$$V = \iiint_T dV$$

$$= \int_{-2}^{2}\int_{0}^{4-x^2}\int_{x^2}^{4-y} dz\,dy\,dx$$

$$= \int_{-2}^{2}\int_{0}^{4-x^2} z\Big|_{z=x^2}^{z=4-y} dy\,dx$$

Continued →

(17) Cont'd

$$= \int_{-2}^{2} \int_{0}^{4-x^2} (4-y-x^2)\, dy\, dx$$

$$= \int_{-2}^{2} \left[(4-x^2)y - \frac{y^2}{2} \right]_{y=0}^{y=4-x^2} dx$$

$$= \int_{-2}^{2} \left[(4-x^2)(4-x^2) - \frac{1}{2}(4-x^2)^2 \right] dx$$

$$= \int_{-2}^{2} \frac{1}{2}(4-x^2)^2\, dx$$

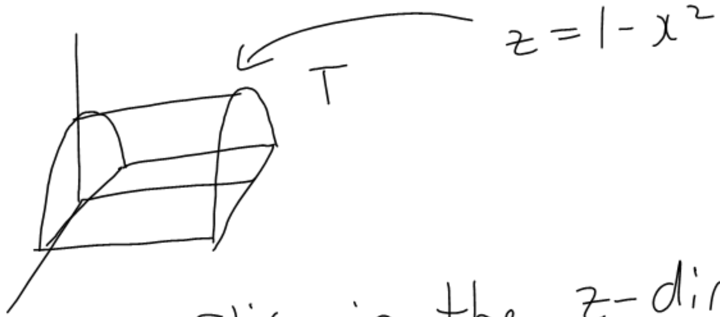
$$= \frac{1}{2} \int_{-2}^{2} (16 - 8x^2 + x^4)\, dx$$

$$= \frac{1}{2} \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_{-2}^{2}$$

$$= \frac{1}{2} \left[32 - \frac{64}{3} + \frac{32}{5} - \left(-32 + \frac{64}{3} - \frac{32}{5} \right) \right]$$

$$= \frac{256}{15}$$

(27)



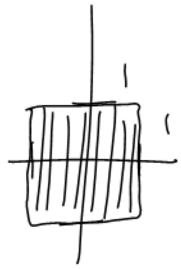
slice in the z-direction:

$$0 \le z \le 1 - x^2$$

Project on the xy-plane:

$$z = z$$
$$1 - x^2 = 0$$
$$1 = x^2$$
$$x = \pm 1$$
and $y = \pm 1$

R :



R : $-1 \le y \le 1$

$-1 \le x \le 1$

$$I_y = \iiint_T (x^2 + z^2)\,\delta\, dV$$

Given $\delta = 1$

$$= \int_{-1}^{1}\int_{-1}^{1}\int_{0}^{1-x^2} (x^2 + z^2)\, dz\, dy\, dx$$

$$= \int_{-1}^{1}\int_{-1}^{1} \left[x^2 z + \frac{z^3}{3} \right]_{z=0}^{z=1-x^2} dy\, dx$$

$$= \int_{-1}^{1}\int_{-1}^{1} \left[x^2(1-x^2) + \frac{1}{3}(1-x^2)^3 \right] dy\, dx$$

Continued $\longrightarrow$

(27) Cont'd

$$= \int_{-1}^{1} \left[x^2(1-x^2) + \frac{1}{3}(1-x^2)^3\right] y \Big|_{y=-1}^{y=1} dx$$

$$= \int_{-1}^{1} \left[x^2(1-x^2) + \frac{1}{3}(1-x^2)^3\right] (1-(-1)) \, dx$$

$$= 2\int_{-1}^{1} \left[x^2 - x^4 + \frac{1}{3}(1-x^2)(1-2x^2+x^4)\right] dx$$

$$= 2\int_{-1}^{1} \left[x^2 - x^4 + \frac{1}{3}(1-2x^2+x^4-x^2+2x^4-x^6)\right] dx$$

$$= 2\int_{-1}^{1} \left[x^2 - x^4 + \frac{1}{3}(1-3x^2+3x^4-x^6)\right] dx$$

$$= 2\int_{-1}^{1} \left(\frac{1}{3} - \frac{x^6}{3}\right) dx$$

$$= 2\left[\frac{x}{3} - \frac{x^7}{21}\right]_{-1}^{1}$$

$$= 2\left[\frac{1}{3} - \frac{1}{21} - \left(\frac{-1}{3} + \frac{1}{21}\right)\right]$$

$$= \frac{8}{7}$$