(3)

Intersection

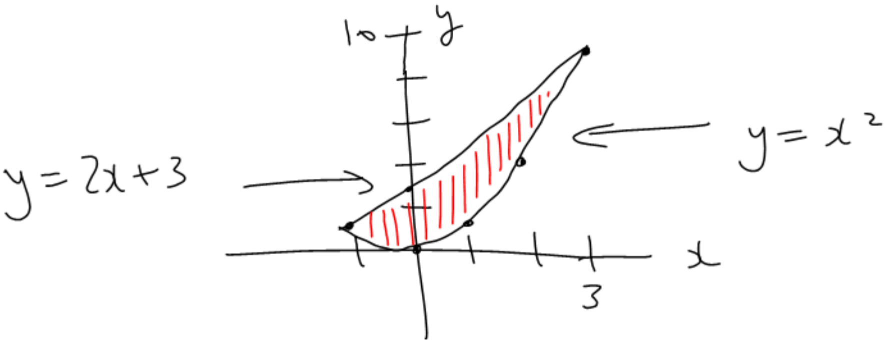
$$y = y$$

$$x^2 = 2x + 3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = -1, 3$$



$$R: \quad x^2 \leq y \leq 2x+3$$

$$-1 \leq x \leq 3$$

$$A = \int_{-1}^{3} \int_{x^2}^{2x+3} dy \, dx$$

$$= \int_{-1}^{3} y \Big|_{y=x^2}^{y=2x+3} dx$$

$$= \int_{-1}^{3} (2x+3-x^2)\, dx$$

$$= \left[x^2 + 3x - \frac{x^3}{3} \right]_{-1}^{3}$$

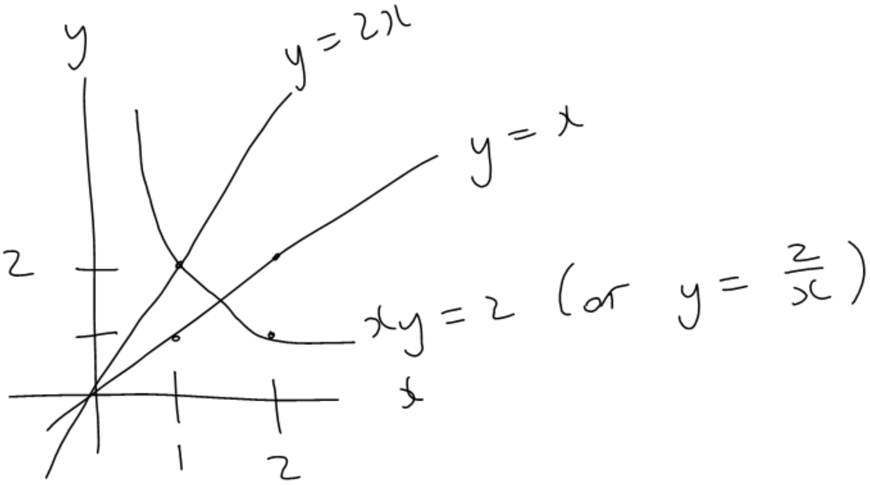
Continued $\rightarrow$

3 Cont'd

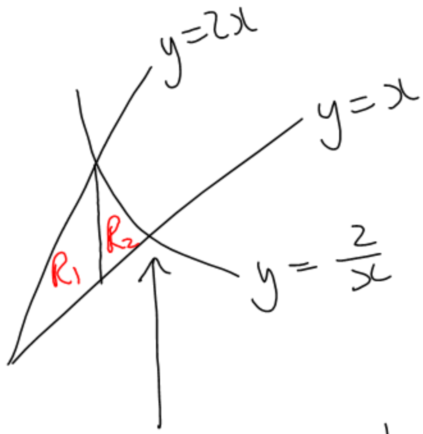
$$= 9 + 9 - \frac{27}{3} - 1 + 3 - \frac{1}{3}$$

$$= \frac{32}{3}$$

(9)



We must break R into two regions:



Need this x-value.

Intersection:

$$y = y$$

$$x = \frac{2}{x}$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$x = \sqrt{2}$$

$$\boxed{R_1: \quad x \le y \le 2x \qquad 0 \le x \le 1}$$

$$\boxed{R_2: \quad x \le y \le \frac{2}{x} \qquad 1 \le x \le \sqrt{2}}$$

Continued →

⑨ Cont'd

$$A_1 = \int_0^1 \int_x^{2x} dy\, dx$$

$$= \int_0^1 y \Big|_{y=x}^{y=2x} dx$$

$$= \int_0^1 (\underbrace{\cancel{2x - x}}_{x})\, dx$$

$$= \frac{x^2}{2} \Big|_0^1$$

$$= \frac{1}{2}$$

$$A_2 = \int_1^{\sqrt{2}} \int_x^{2/x} dy\, dx$$

$$= \int_1^{\sqrt{2}} y \Big|_{y=x}^{y=\frac{2}{x}} dx$$

$$= \int_1^{\sqrt{2}} \left(\frac{2}{x} - x\right) dx$$

$$= \left[2\ln|x| - \frac{x^2}{2}\right]_1^{\sqrt{2}}$$

$$= 2\ln\sqrt{2} - \frac{2}{2} + \frac{1}{2}$$

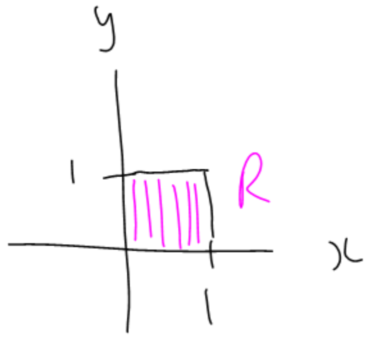
$$= 2\ln\sqrt{2} - \frac{1}{2}$$

$$= \ln\sqrt{2}^2 - \frac{1}{2}$$

$$= \ln 2 - \frac{1}{2}$$

$$\boxed{A = A_1 + A_2 = \ln 2}$$

(11)



$$R: \quad \begin{aligned} 0 \le y \le 1 \\ 0 \le x \le 1 \end{aligned}$$

$$V = \iint_R z \, dA$$

$$= \int_0^1 \int_0^1 (1 + x + y) \, dy \, dx$$

$$= \int_0^1 \left[y + xy + \frac{y^2}{2} \right]_{y=0}^{y=1} dx$$

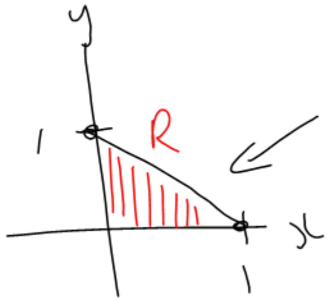
$$= \int_0^1 \left(1 + x + \frac{1}{2}\right) dx$$

$$= \int_0^1 \left(\frac{3}{2} + x\right) dx$$

$$= \left[\frac{3}{2}x + \frac{x^2}{2} \right]_0^1$$

$$= 2$$

(15)



$x + y = 1$ (or $y = 1 - x$)

$$R: \quad 0 \le y \le 1 - x$$
$$\quad\;\; 0 \le x \le 1$$

$$V = \iint_R z \, dA$$

$$= \int_0^1 \int_0^{1-x} (x+y) \, dy \, dx$$

$$= \int_0^1 \left[xy + \frac{y^2}{2} \right]_{y=0}^{y=1-x} dx$$

$$= \int_0^1 \left[x(1-x) + \frac{1}{2}(1-x)^2 \right] dx$$

$$x(1-x) + \frac{1}{2}(1-x)^2$$
$$= x - x^2 + \frac{1}{2}(1 - 2x + x^2)$$
$$= x - x^2 + \frac{1}{2} - x + \frac{x^2}{2}$$
$$= -\frac{x^2}{2} + \frac{1}{2}$$

$$= \int_0^1 \left(-\frac{x^2}{2} + \frac{1}{2} \right) dx$$

Continued →

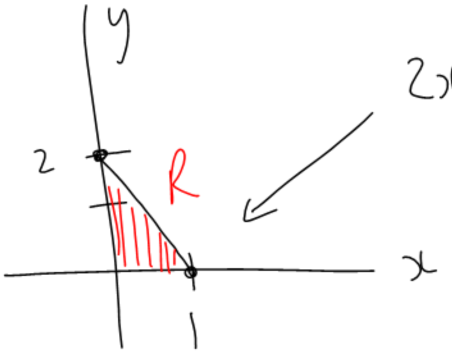
15 Cont'd

$$= \left[-\frac{x^3}{6} + \frac{x}{2} \right]_0^1$$

$$= -\frac{1}{6} + \frac{1}{2}$$

$$= \frac{1}{3}$$

25



$2x + y = 2$ (or $y = 2 - 2x$)

$R: \quad 0 \le y \le 2 - 2x$

$\qquad 0 \le x \le 1$

$$V = \iint_R z \, dA$$

$$= \int_0^1 \int_0^{2-2x} (4x^2 + y^2) \, dy \, dx$$

$$= \int_0^1 \left[4x^2 y + \frac{y^3}{3} \right]_{y=0}^{y=2-2x} dx$$

$$= \int_0^1 \left[4x^2(2-2x) + \frac{1}{3}(2-2x)^3 \right] dx$$

$$= \underbrace{\int_0^1 (8x^2 - 8x^3) \, dx}_{I_1} + \frac{1}{3} \underbrace{\int_0^1 (2-2x)^3 \, dx}_{I_2}$$

Continued →

(25) Cont'd

$$I_1 = \left[\frac{8x^3}{3} - 2x^4\right]_0^1$$

$$= \frac{8}{3} - 2$$

$$= \frac{2}{3}$$

For I_2, let

$$u = 2 - 2x$$
$$du = -2dx$$
$$-\frac{1}{2}du = dx$$
$$x = 0 \Rightarrow u = 2$$
$$x = 1 \Rightarrow u = 0$$

$$I_2 = -\frac{1}{6}\int_2^0 u^3\, du$$

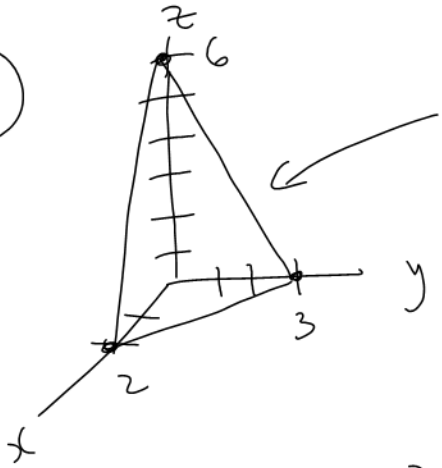
$$= -\frac{1}{24} u^4 \Big|_2^0$$

$$= -\frac{1}{24}(-16)$$

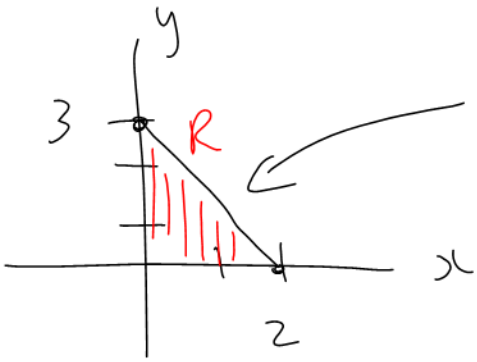
$$= \frac{2}{3}$$

$$V = I_1 + I_2$$
$$= \frac{4}{3}$$

(27)



$3x+2y+z=6$
(or $z=6-3x-2y$)



$y = mx+b$
$y = 3-\frac{3}{2}x$

R: $0 \le y \le 3-\frac{3}{2}x$
$0 \le x \le 2$

$$V = \iint_R z \, dA$$

$$= \int_0^2 \int_0^{3-\frac{3}{2}x} (6-3x-2y) \, dy \, dx$$

$$= \int_0^2 \left[6y - 3xy - y^2 \right]_{y=0}^{y=3-\frac{3}{2}x} dx$$

$$= \int_0^2 \left[6\left(3-\tfrac{3}{2}x\right) - 3x\left(3-\tfrac{3}{2}x\right) - \left(3-\tfrac{3}{2}x\right)^2 \right] dx$$

Continued $\longrightarrow$

(27) Cont'd

$$\boxed{\begin{aligned}&6\left(3-\tfrac{3}{2}x\right)-3x\left(3-\tfrac{3}{2}x\right)-\left(3-\tfrac{3}{2}x\right)^2\\&=18-9x-9x+\frac{9x^2}{2}-\left(9-9x+\frac{9}{4}x^2\right)\\&=18-18x+\frac{9x^2}{2}-9+9x-\frac{9}{4}x^2\\&=9-9x+\frac{9x^2}{4}\end{aligned}}$$

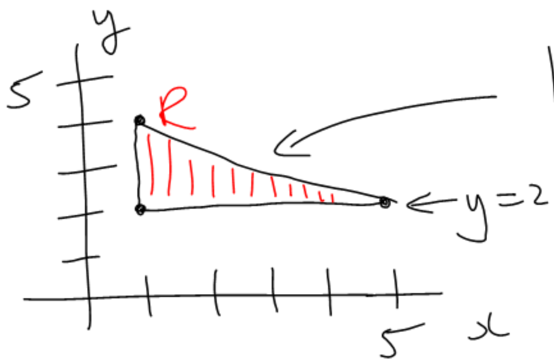
$$= \int_0^2 \left(9-9x+\frac{9x^2}{4}\right)dx$$

$$= \left[9x-\frac{9x^2}{2}+\frac{3x^3}{4}\right]_0^2$$

$$= 18-18+6$$

$$= 6$$

(29)



$$m = -\frac{2}{4}$$
$$= -\frac{1}{2}$$
$$y = mx + b$$
$$y = -\frac{x}{2} + b$$

Sub $(x, y) = (1, 4)$:
$$4 = \frac{-1}{2} + b$$
$$\frac{9}{2} = b$$
$$y = \frac{-x}{2} + \frac{9}{2}$$

$$R: \quad 2 \le y \le \frac{9}{2} - \frac{x}{2}$$
$$1 \le x \le 5$$

$$V = \iint_R z \, dA$$
$$= \int_1^5 \int_2^{\frac{9}{2} - \frac{x}{2}} xy \, dy \, dx$$
$$= \int_1^5 \left. \frac{xy^2}{2} \right|_{y=2}^{y = \frac{9}{2} - \frac{x}{2}} dx$$
$$= \int_1^5 \left[\frac{x}{2} \left(\frac{9}{2} - \frac{x}{2} \right)^2 - 2x \right] dx$$

Continued →

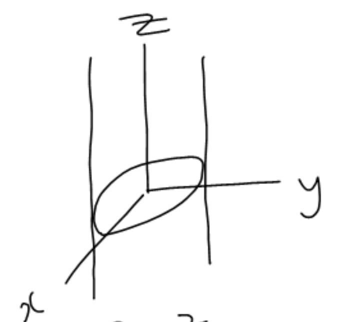
29 Cont'd

$$\frac{x}{2}\left(\frac{9}{2}-\frac{x}{2}\right)^2-2x$$
$$=\frac{x}{2}\left(\frac{81}{4}-\frac{9x}{2}+\frac{x^2}{4}\right)-2x$$
$$=\frac{81x}{8}-\frac{9x^2}{4}+\frac{x^3}{8}-2x$$
$$=\frac{65x}{8}-\frac{9x^2}{4}+\frac{x^3}{8}$$

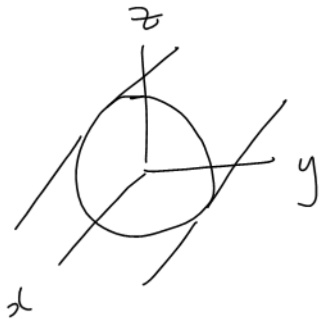
$$=\int_1^5\left[\frac{65x}{8}-\frac{9x^2}{4}+\frac{x^3}{8}\right]dx$$
$$=\left[\frac{65x^2}{16}-\frac{3x^3}{4}+\frac{x^4}{32}\right]_1^5$$
$$=\frac{1625}{16}-\frac{375}{4}+\frac{625}{32}-\frac{65}{16}+\frac{3}{4}-\frac{1}{32}$$

$$=24$$

(37)



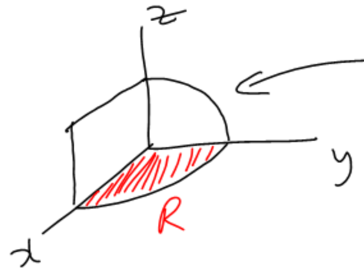
$x^2+y^2=1$



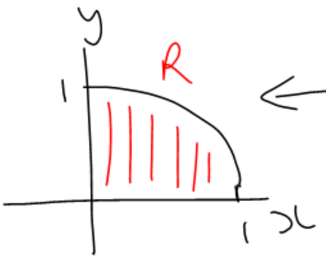
$y^2+z^2=1$

The solid is restricted to the first octant (meaning $x, y, z > 0$).

The solid:



$$y^2+z^2=1$$
$$z^2=1-y^2$$
$$z=\pm\sqrt{1-y^2}$$
$$z=\sqrt{1-y^2}$$



$$x^2+y^2=1$$
$$y^2=1-x^2$$
$$y=\pm\sqrt{1-x^2}$$
$$y=\sqrt{1-x^2}$$

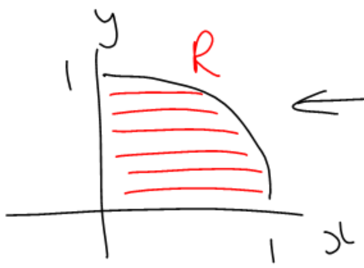
$$R: \quad 0 \le y \le \sqrt{1-x^2}$$
$$0 \le x \le 1$$

$$V = \iint_R z \, dA$$
$$= \int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2} \; dy \, dx$$

This is difficult to integrate. Rewrite R with horizontal slices.

Continued $\longrightarrow$

37 Cont'd



$$x^2+y^2=1$$
$$x^2=1-y^2$$
$$x=\pm\sqrt{1-y^2}$$
$$x=\sqrt{1-y^2}$$

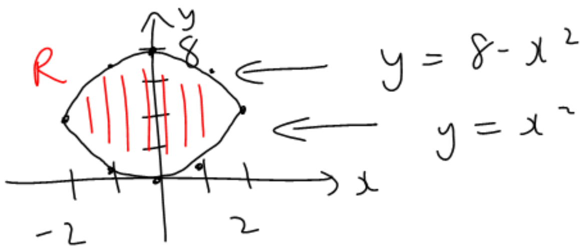
$$R: \quad 0 \le x \le \sqrt{1-y^2}$$
$$0 \le y \le 1$$

$$V = \iint_R z \, dA$$
$$= \int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-y^2} \, dx \, dy$$
$$= \int_0^1 \sqrt{1-y^2} \, x \Big|_{x=0}^{x=\sqrt{1-y^2}} \, dy$$
$$= \int_0^1 (1-y^2) \, dy$$
$$= \left[y - \frac{y^3}{3} \right]_0^1$$
$$= \frac{2}{3}$$

45

Intersection:

$$y = y$$
$$x^2 = 8 - x^2$$
$$2x^2 = 8$$
$$x^2 = 4$$
$$x = \pm 2$$



$$R: \quad x^2 \le y \le 8 - x^2$$
$$-2 \le x \le 2$$

$$V = \iint_R (z_{top} - z_{bottom})\, dA$$

Since $2x^2 \geq x^2$ for all points in R,
$z_{top} = 2x^2$ and $z_{bottom} = x^2$

$$V = \int_{-2}^{2} \int_{x^2}^{8-x^2} (2x^2 - x^2)\, dy\, dx$$

$$= \int_{-2}^{2} \int_{x^2}^{8-x^2} x^2\, dy\, dx$$

$$= \int_{-2}^{2} x^2 y \Big|_{y=x^2}^{y=8-x^2} dx$$

Continued $\longrightarrow$

45 Cont'd

$$= \int_{-2}^{2} \left[x^2(8-x^2) - x^4\right] dx$$

$$= \int_{-2}^{2} (8x^2 - 2x^4)\, dx$$

$$= \left[\frac{8x^3}{3} - \frac{2x^5}{5}\right]_{-2}^{2}$$

$$= \frac{64}{3} - \frac{64}{5} + \frac{64}{3} - \frac{64}{5}$$

$$= \frac{256}{15}$$