(7) $$\int_0^1 \int_x^{\sqrt{x}} (2x-y)\, dy\, dx$$

$$= \int_0^1 \left[2xy - \frac{y^2}{2} \right]_{y=x}^{y=\sqrt{x}} dx$$

$$= \int_0^1 \left[2x^{3/2} - \frac{x}{2} - 2x^2 + \frac{x^2}{2} \right] dx$$

$$= \left[\frac{4}{5} x^{5/2} - \frac{x^2}{4} - \frac{2x^3}{3} + \frac{x^3}{6} \right]_0^1$$

$$= \frac{4}{5} - \frac{1}{4} - \frac{2}{3} + \frac{1}{6}$$

$$= \frac{3}{60}$$

$$= \frac{1}{20}$$

(11)

$$\int_0^1 \int_0^{x^3} e^{y/x} \, dy \, dx$$

$$= \int_0^1 \left[x e^{y/x} \right]_{y=0}^{y=x^3} dx$$

Recall $\int e^{ky} \, dy = \frac{1}{k} e^{ky} + C$

$k = \frac{1}{x}$ implies

$\int e^{y/x} \, dy = x e^{y/x} + C$

$$= \int_0^1 \left(x e^{x^2} - x \right) dx$$

$I = \int x e^{x^2} \, dx$

Let $u = x^2$

$du = 2x \, dx$

$\frac{du}{2} = x \, dx$

$I = \frac{1}{2} \int e^u \, du$

$= \frac{1}{2} e^u + C$

$= \frac{1}{2} e^{x^2} + C$

$$= \left[\frac{1}{2} e^{x^2} - \frac{x^2}{2} \right]_0^1$$

$$= \frac{1}{2} e - \frac{1}{2} - \frac{1}{2}$$

$$= \frac{e}{2} - 1$$

(13)

$$\int_0^3 \int_0^y \sqrt{y^2+16}\, dx\, dy$$

$$= \int_0^3 \left[\sqrt{y^2+16}\; x \right]_{x=0}^{x=y} dy$$

$$= \int_0^3 \sqrt{y^2+16}\; y\, dy$$

$$\boxed{\begin{aligned} &\text{Let } u = y^2+16 \\ &du = 2y\,dy \\ &\frac{du}{2} = y\,dy \\ &y=0 \Rightarrow u=16 \\ &y=3 \Rightarrow u=25 \end{aligned}}$$

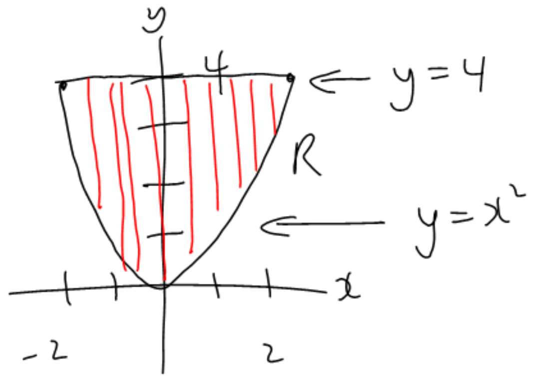
$$= \frac{1}{2} \int_{16}^{25} \sqrt{u}\, du$$

$$= \frac{1}{2}\, \frac{2}{3}\, u^{3/2} \Big|_{16}^{25}$$

$$= \frac{1}{3}(125 - 64)$$

$$= \frac{61}{3}$$

(15)



Intersection:

$$y = y$$
$$x^2 = 4$$
$$x = \pm 2$$

$$\boxed{R: \quad \begin{array}{c} x^2 \le y \le 4 \\ -2 \le x \le 2 \end{array}}$$

$$\iint_R f \, dA$$

$$= \int_{-2}^{2} \int_{x^2}^{4} xy \, dy \, dx$$

$$= \int_{-2}^{2} \left. \frac{xy^2}{2} \right|_{y=x^2}^{y=4} dx$$

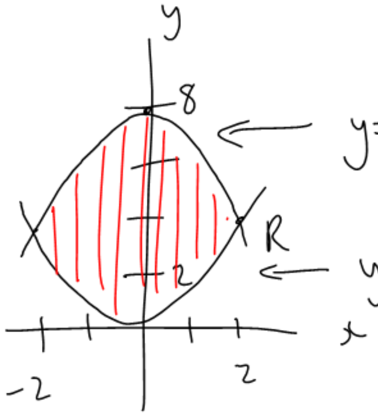
$$= \int_{-2}^{2} \left[8x - \frac{x^5}{2} \right] dx$$

$$= \left[4x^2 - \frac{x^6}{12} \right]_{-2}^{2}$$

$$= 16 - \frac{64}{12} - 16 + \frac{64}{12}$$

$$= 0$$

(17)



$y = 8 - x^2$

$y = x^2$

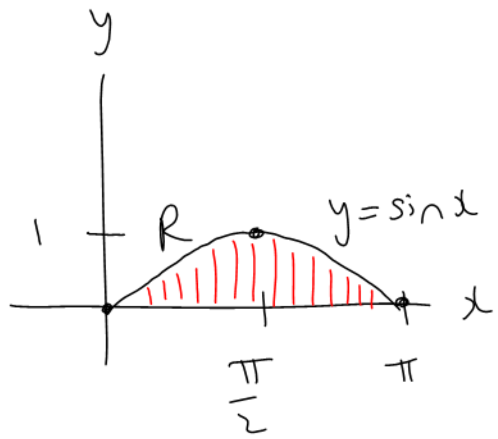
Intersection:

$$y = y$$
$$x^2 = 8 - x^2$$
$$2x^2 = 8$$
$$x^2 = 4$$
$$x = \pm 2$$

$$\boxed{R: \quad \begin{matrix} x^2 \le y \le 8 - x^2 \\ -2 \le x \le 2 \end{matrix}}$$

$$\iint_R f \, dA$$
$$= \int_{-2}^{2} \int_{x^2}^{8-x^2} x \; dy \, dx$$
$$= \int_{-2}^{2} xy \Big|_{y=x^2}^{y=8-x^2} dx$$
$$= \int_{-2}^{2} (8x - x^3 - x^3) \, dx$$
$$= \left[4x^2 - \frac{x^4}{2} \right]_{-2}^{2}$$
$$= 16 - 8 - 16 + 8$$
$$= 0$$

19



$$\boxed{R: \begin{array}{l} 0 \le y \le \sin x \\ 0 \le x \le \pi \end{array}}$$

$$\iint_R f \, dA$$

$$= \int_0^{\pi} \int_0^{\sin x} x \, dy \, dx$$

$$= \int_0^{\pi} xy \Big|_{y=0}^{y=\sin x} dx$$

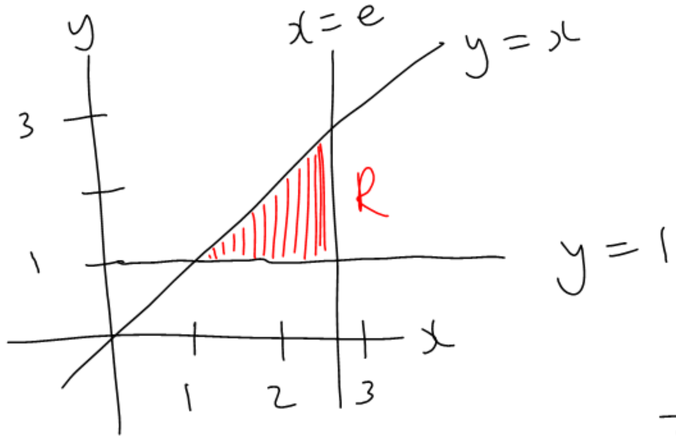
$$= \int_0^{\pi} x \sin x \, dx$$

	D	I
⊕	x	$\sin x$
⊖	1	$-\cos x$
		$-\sin x$

$$= \Big[-x\cos x + \sin x \Big]_0^{\pi}$$

$$= \pi$$

(21)



$e \simeq 2.7$

$$R: \quad \begin{array}{l} 1 \le y \le x \\ 1 \le x \le e \end{array}$$

$$\iint_R f \, dA$$

$$= \int_1^e \int_1^x \frac{1}{y} \, dy \, dx$$

$$= \int_1^e \Big[\ln|y| \Big]_{y=1}^{y=x} dx$$

$$= \int_1^e \ln x \, dx$$

We can drop absolute value because $x > 0$.

Integration by Parts

$u = \ln x \quad dv = 1 \, dx$

$du = \frac{1}{x} dx \quad v = x$

$\int u \, dv = uv - \int v \, du$

$\int \ln x \, dx = x \ln x - \int dx$

$= x \ln x - x + C$

Continued $\longrightarrow$

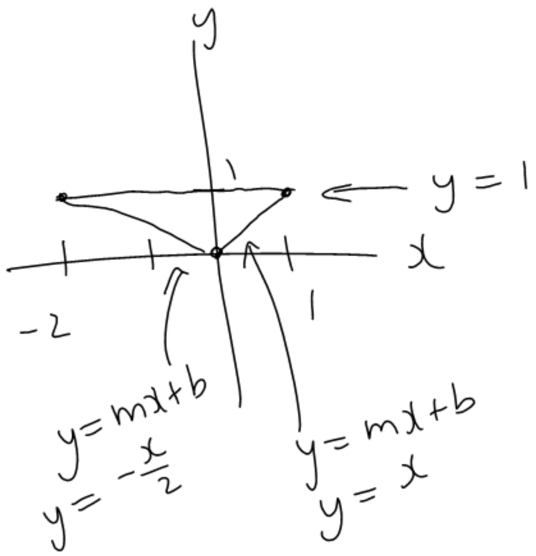
21 Cont'd

$$= \left[x\ln x - x\right]_1^e$$

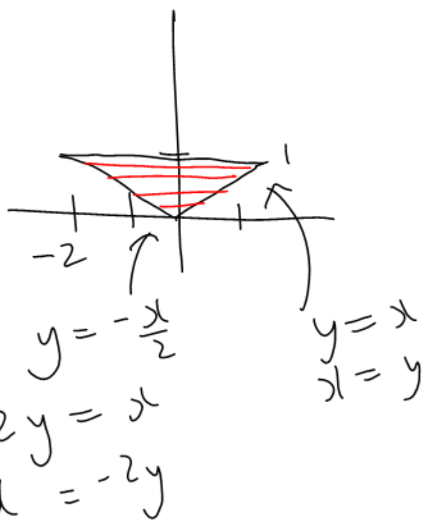
$$= e - e - 0 + 1$$

$$= 1$$

(23)



Horizontal slices are easier.



$$\boxed{R: \begin{array}{l} -2y \le x \le y \\ 0 \le y \le 1 \end{array}}$$

$$\iint f \, dA$$

$$= \int_0^1 \int_{-2y}^{y} (1-x)\, dx\, dy$$

$$= \int_0^1 \left[x - \frac{x^2}{2} \right]_{x=-2y}^{x=y} dy$$

Continued →

23 Cont'd

$$= \int_0^1 \left[y - \frac{y^2}{2} + 2y + 2y^2 \right] dy$$

$$= \left[\frac{y^2}{2} - \frac{y^3}{6} + y^2 + \frac{2y^3}{3} \right]_0^1$$

$$= \frac{1}{2} - \frac{1}{6} + 1 + \frac{2}{3}$$

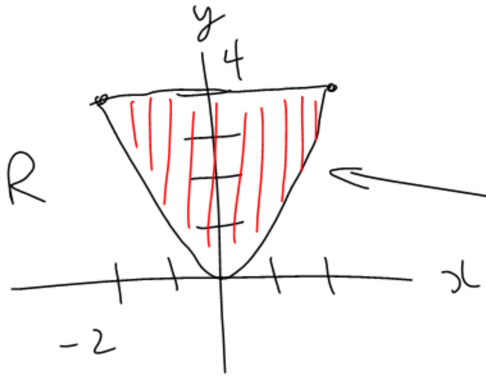
$$= \frac{12}{6}$$

$$= 2$$

25

$R: \quad x^2 \le y \le 4$

$-2 \le x \le 2$

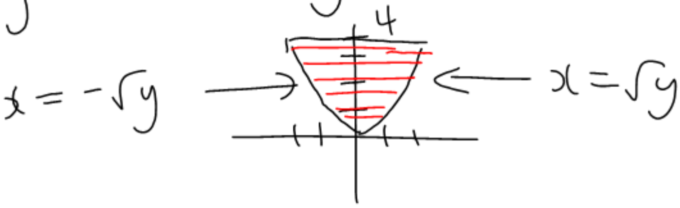


$y = x^2$

$x^2 = y$

$x = \pm\sqrt{y}$

Note: $x = -\sqrt{y}$ is the left half of the parabola.
$x = \sqrt{y}$ is the right half of the parabola.



$$\boxed{\begin{aligned} R: \ -\sqrt{y} \le x \le \sqrt{y} \\ 0 \le y \le 4 \end{aligned}}$$

$$\text{Integral} = \int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} x^2 y \, dx \, dy$$

$$= \int_0^4 \left. \frac{x^3 y}{3} \right|_{x=-\sqrt{y}}^{x=\sqrt{y}} dy$$

$$= \int_0^4 \left[\frac{y^{5/2}}{3} + \frac{y^{5/2}}{3} \right] dy$$

Continued $\longrightarrow$

25 Cont'd

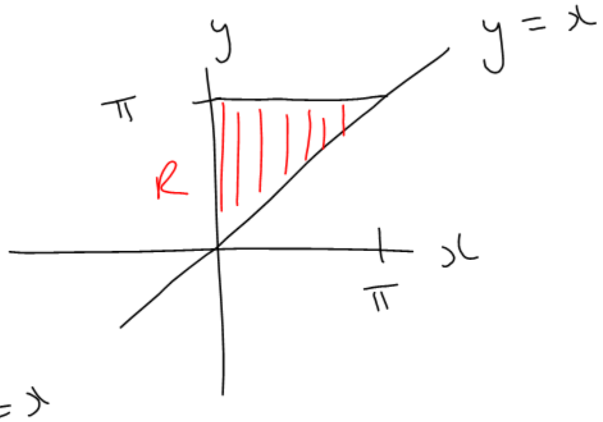
$$= \frac{2}{3}\left(\frac{2}{7} y^{7/2}\right)\Big|_0^4$$

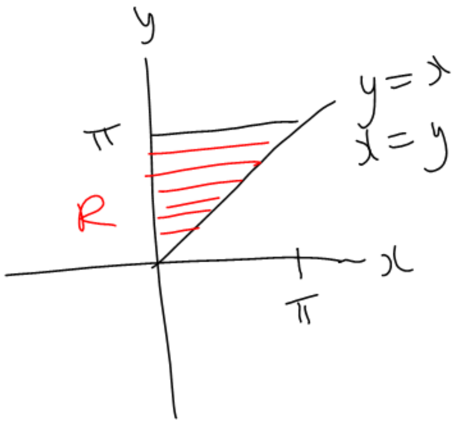
$$= \frac{4}{21}(128)$$

$$= \frac{512}{21}$$

(31) R: $x \le y \le \pi$

$0 \le x \le \pi$





R: $0 \le x \le y$

$0 \le y \le \pi$

$$\text{Integral} = \int_0^{\pi}\int_0^{y} \frac{\sin y}{y}\, dx\, dy$$

$$= \int_0^{\pi} \left[\frac{\sin y}{y} x \right]_{x=0}^{x=y} dy$$

$$= \int_0^{\pi} \sin y \, dy$$

$$= \left[-\cos y \right]_0^{\pi}$$

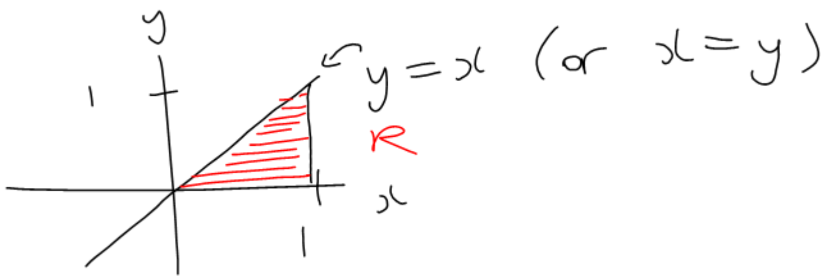
$$= 1 + 1$$

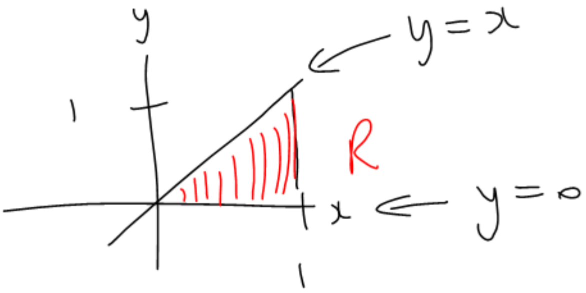
$$= 2$$

(33)

$R:\quad y \le x \le 1$

$\quad\;\; 0 \le y \le 1$





$R:\quad 0 \le y \le x$

$\quad\;\; 0 \le x \le 1$

$$\text{Integral} = \int_0^1 \int_0^x \frac{1}{1+x^4}\, dy\, dx$$

$$= \int_0^1 \left[\frac{1}{1+x^4}\, y \right]_{y=0}^{y=x} dx$$

$$= \int_0^1 \frac{x}{1+x^4}\, dx$$

$$= \int_0^1 \frac{x}{1+(x^2)^2}\, dx$$

Sub $u = x^2$

$du = 2x\,dx$

$\dfrac{du}{2} = x\,dx$

$x = 0 \Rightarrow u = 0$

$x = 1 \Rightarrow u = 1$

Continued →

(33) Cont'd

$$= \frac{1}{2} \int_0^1 \frac{du}{1+u^2}$$

$$= \frac{1}{2} \left[\arctan u\right]_0^1$$

$$= \frac{1}{2} \left[\arctan 1 - \arctan 0\right]$$

$$= \frac{1}{2} \left[\frac{\pi}{4}\right]$$

$$= \frac{\pi}{8}$$