

⑪

$$\int_0^2 \int_0^4 (3x + 4y) dx dy$$

$$= \int_0^2 \left[\frac{3x^2}{2} + 4xy \right]_{x=0}^{x=4} dy$$

$$= \int_0^2 [24 + 16y - 0] dy$$

$$= [24y + 8y^2]_0^2$$

$$= 48 + 32 - 0$$

$$= 80$$

(15)

$$\int_0^3 \int_0^3 (xy + 7x + y) dx dy$$

$$= \int_0^3 \left[\frac{x^2 y}{2} + \frac{7x^2}{2} + xy \right]_{x=0}^{x=3} dy$$

$$= \int_0^3 \left[\frac{9y}{2} + \frac{63}{2} + 3y - 0 \right] dy$$

$$= \left[\frac{9y^2}{4} + \frac{63}{2}y + \frac{3y^2}{2} \right]_0^3$$

$$= \frac{81}{4} + \frac{189}{2} + \frac{27}{2}$$

$$= \frac{81}{4} + \frac{378}{4} + \frac{54}{4}$$

$$= \frac{513}{4}$$

(17)

$$\int_{-1}^2 \int_{-1}^2 (2xy^2 - 3x^2y) dy dx$$

$$= \int_{-1}^2 \left[\frac{2xy^3}{3} - \frac{3x^2y^2}{2} \right]_{y=-1}^{y=2} dx$$

$$= \int_{-1}^2 \left[\frac{16x}{3} - 6x^2 - \left(-\frac{2x}{3} - \frac{3x^2}{2} \right) \right] dx$$

$$= \int_{-1}^2 \left[\frac{16x}{3} - 6x^2 + \frac{2x}{3} + \frac{3x^2}{2} \right] dx$$

$$= \left[\frac{16x^2}{6} - 2x^3 + \frac{x^2}{3} + \frac{x^3}{2} \right]_{-1}^2$$

$$= \frac{64}{6} - 16 + \frac{4}{3} + 4 - \left(\frac{16}{6} + 2 + \frac{1}{3} - \frac{1}{2} \right)$$

$$= \frac{64}{6} - \frac{96}{6} + \frac{8}{6} + \frac{24}{6} - \frac{16}{6} - \frac{12}{6} - \frac{2}{6} + \frac{3}{6}$$

$$= -\frac{27}{6}$$

$$= -\frac{9}{2}$$

(27)

$$\int_0^{\frac{\pi}{2}} \int_1^e \frac{\sin y}{x} dx dy$$

$$= \int_0^{\frac{\pi}{2}} \left[(\sin y) \ln |x| \right]_{x=1}^{x=e} dy$$

$$= \int_0^{\frac{\pi}{2}} [\sin y - 0] dy$$

$$= [-\cos y]_0^{\frac{\pi}{2}}$$

$$= 0 + 1$$

$$= 1$$