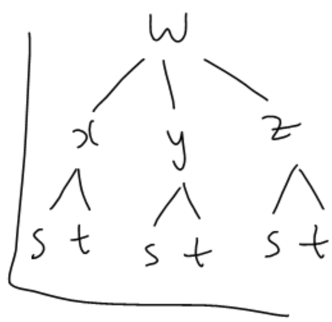
(5) $w = \ln(x^2+y^2+z^2)$, $x = s-t$, $y = s+t$,

$z = 2\sqrt{st}$



$$\frac{\partial w}{\partial x} = \frac{1}{x^2+y^2+z^2}(2x) = \frac{2x}{x^2+y^2+z^2}$$

Similarly, $\frac{\partial w}{\partial y} = \frac{2y}{x^2+y^2+z^2}$ and $\frac{\partial w}{\partial z} = \frac{2z}{x^2+y^2+z^2}$

$$\frac{\partial x}{\partial s} = 1 \qquad \frac{\partial x}{\partial t} = -1$$

$$\frac{\partial y}{\partial s} = 1 \qquad \frac{\partial y}{\partial t} = 1$$

$$z = 2\sqrt{s}\sqrt{t}$$

$$\frac{\partial z}{\partial s} = 2\sqrt{t}\left(\frac{1}{2}s^{-1/2}\right) = \frac{\sqrt{t}}{\sqrt{s}}$$

$$\frac{\partial z}{\partial t} = 2\sqrt{s}\left(\frac{1}{2}t^{-1/2}\right) = \frac{\sqrt{s}}{\sqrt{t}}$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

$$= \frac{2x}{x^2+y^2+z^2}(1) + \frac{2y}{x^2+y^2+z^2}(1) + \frac{2z}{x^2+y^2+z^2}\left(\frac{\sqrt{t}}{\sqrt{s}}\right)$$

No need to simplify.

Continued →

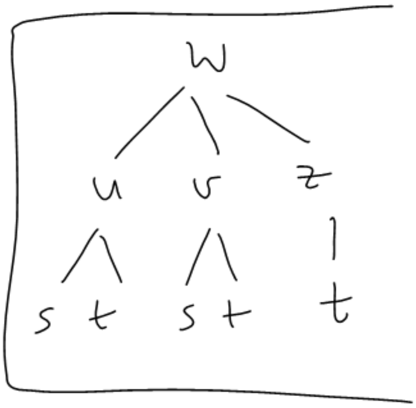
5 Cont'd

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$= \frac{2x}{x^2+y^2+z^2}(-1) + \frac{2y}{x^2+y^2+z^2}(1) + \frac{2z}{x^2+y^2+z^2}\left(\frac{\sqrt{5}}{\sqrt{t}}\right)$$

No need to simplify.

(7) $w = \sqrt{u^2+v^2+z^2}$, $u = 3e^t \sin s$, $v = 3e^t \cos s$, $z = 4e^t$



$$\frac{\partial w}{\partial u} = \frac{1}{2}\left(u^2+v^2+z^2\right)^{-1/2}(2u)$$
$$= \frac{u}{\sqrt{u^2+v^2+z^2}}$$

Similarly, $\frac{\partial w}{\partial v} = \frac{v}{\sqrt{u^2+v^2+z^2}}$ and $\frac{\partial w}{\partial z} = \frac{z}{\sqrt{u^2+v^2+z^2}}$

$$\frac{\partial u}{\partial s} = 3e^t \cos s \qquad \frac{\partial u}{\partial t} = 3e^t \sin s$$

$$\frac{\partial v}{\partial s} = -3e^t \sin s \qquad \frac{\partial v}{\partial t} = 3e^t \cos s$$

$$\frac{dz}{dt} = 4e^t$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial u}\frac{\partial u}{\partial s} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial s}$$
$$= \frac{u}{\sqrt{u^2+v^2+z^2}}\left(3e^t \cos s\right) + \frac{v}{\sqrt{u^2+v^2+z^2}}\left(-3e^t \sin s\right)$$

No need to simplify.

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial t} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

Continued →

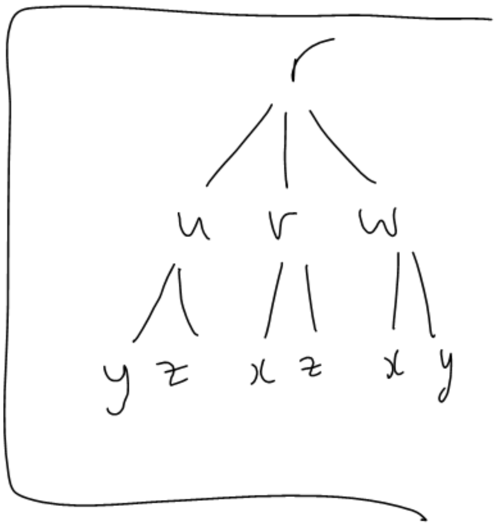
⑦ Cont'd

$$\frac{\partial w}{\partial t} = \frac{u}{\sqrt{u^2+v^2+z^2}}\left(3e^t \sin s\right) + \frac{v}{\sqrt{u^2+v^2+z^2}}\left(3e^t \cos s\right)$$

$$+ \frac{z}{\sqrt{u^2+v^2+z^2}}\left(4e^t\right)$$

No need to simplify.

⑨ $r = e^{u+v+w}$, $u = yz$, $v = xz$, $w = xy$



$$\frac{\partial r}{\partial u} = e^{u+v+w}(1) = e^{u+v+w}$$

Similarly $\frac{\partial r}{\partial v} = e^{u+v+w}$

and $\frac{\partial r}{\partial w} = e^{u+v+w}$

$\frac{\partial u}{\partial y} = z$ $\quad \frac{\partial u}{\partial z} = y$

$\frac{\partial v}{\partial x} = z$ $\quad \frac{\partial v}{\partial z} = x$

$\frac{\partial w}{\partial x} = y$ $\quad \frac{\partial w}{\partial y} = x$

$$\begin{aligned}\frac{\partial r}{\partial x} &= \frac{\partial r}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial r}{\partial w}\frac{\partial w}{\partial x} \\ &= e^{u+v+w}(z) + e^{u+v+w}(y) \\ &= (y+z)e^{u+v+w}\end{aligned}$$

No need to simplify.

Continued →

9 Cnt'd

$$\frac{\partial r}{\partial y} = \frac{\partial r}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial r}{\partial w}\frac{\partial w}{\partial y}$$

$$= e^{u+v+w}(z) + e^{u+v+w}(x)$$

$$= (x+z)\,e^{u+v+w}$$

No need to simplify.

$$\frac{\partial r}{\partial z} = \frac{\partial r}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial r}{\partial v} + \frac{\partial v}{\partial z}$$

$$= e^{u+v+w}(y) + e^{u+v+w}(x)$$

$$= (x+y)\,e^{u+v+w}$$

No need to simplify.

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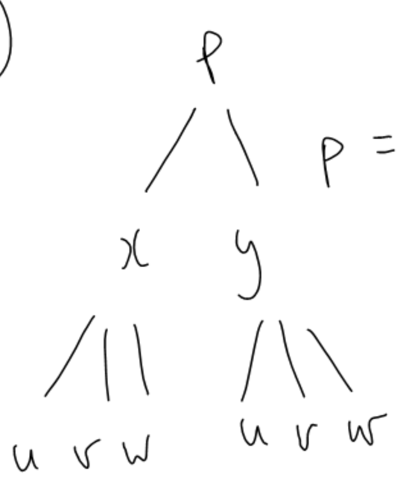

$p = f(x,y)$

The independent variables are u, v, w.

$$\frac{\partial p}{\partial u} = \frac{\partial p}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial p}{\partial y}\frac{\partial y}{\partial u}$$

OR $$\frac{\partial p}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$$

$$\frac{\partial p}{\partial v} = \frac{\partial p}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial p}{\partial y}\frac{\partial y}{\partial v}$$

OR $$\frac{\partial p}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}$$

$$\frac{\partial p}{\partial w} = \frac{\partial p}{\partial x}\frac{\partial x}{\partial w} + \frac{\partial p}{\partial y}\frac{\partial y}{\partial w}$$

OR $$\frac{\partial p}{\partial w} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial w} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial w}$$

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$$\begin{tikzcd} & & p \arrow[d, dash] & & \\ & & w \arrow[dll, dash] \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] \arrow[drr, dash] & & \\ x & y & z & u & v \end{tikzcd}$$

$p = f(w)$

The independent variables are x, y, z, u, v.

$$\frac{\partial p}{\partial x} = \frac{dp}{dw}\frac{\partial w}{\partial x} \quad \text{OR} \quad \frac{\partial p}{\partial x} = \frac{df}{dw}\frac{\partial w}{\partial x}$$

$$\frac{\partial p}{\partial y} = \frac{dp}{dw}\frac{\partial w}{\partial y} \quad \text{OR} \quad \frac{\partial p}{\partial y} = \frac{df}{dw}\frac{\partial w}{\partial y}$$

$$\frac{\partial p}{\partial z} = \frac{dp}{dw}\frac{\partial w}{\partial z} \quad \text{OR} \quad \frac{\partial p}{\partial z} = \frac{df}{dw}\frac{\partial w}{\partial z}$$

$$\frac{\partial p}{\partial u} = \frac{dp}{dw}\frac{\partial w}{\partial u} \quad \text{OR} \quad \frac{\partial p}{\partial u} = \frac{df}{dw}\frac{\partial w}{\partial u}$$

$$\frac{\partial p}{\partial v} = \frac{dp}{dw}\frac{\partial w}{\partial v} \quad \text{OR} \quad \frac{\partial p}{\partial v} = \frac{df}{dw}\frac{\partial w}{\partial v}$$

(21) $xe^{xy} + ye^{zx} + ze^{xy} = 3$

Recall Product Rule: $[uv]' = uv' + vu'$

Take $\frac{\partial}{\partial x}$:

$$x\left[ye^{xy}\right] + e^{xy}(1) + ye^{zx}\left[z(1) + x\frac{\partial z}{\partial x}\right]$$

$$+ z\left[ye^{xy}\right] + e^{xy}\frac{\partial z}{\partial x} = 0$$

$$xye^{xy} + e^{xy} + yze^{zx} + xye^{zx}\frac{\partial z}{\partial x}$$

$$+ yze^{xy} + e^{xy}\frac{\partial z}{\partial x} = 0$$

$$xye^{zx}\frac{\partial z}{\partial x} + e^{xy}\frac{\partial z}{\partial x} = -xye^{xy} - e^{xy} - yze^{zx} - yze^{xy}$$

$$\left[xye^{zx} + e^{xy}\right]\frac{\partial z}{\partial x} = -\left(xye^{xy} + e^{xy} + yze^{zx} + yze^{xy}\right)$$

$$\frac{\partial z}{\partial x} = -\frac{\left(xye^{xy} + e^{xy} + yze^{zx} + yze^{xy}\right)}{xye^{zx} + e^{xy}}$$

Continued →

(21) Cont'd

$$xe^{xy} + ye^{zx} + ze^{xy} = 3$$

Take $\frac{\partial}{\partial y}$:

$$x\left[xe^{xy}\right] + y\left[e^{zx}\, x\frac{\partial z}{\partial y}\right] + e^{zx}(1)$$
$$+ z\left[xe^{xy}\right] + e^{xy}\frac{\partial z}{\partial y} = 0$$

$$x^2e^{xy} + xye^{zx}\frac{\partial z}{\partial y} + e^{zx} + xze^{xy} + e^{xy}\frac{\partial z}{\partial y} = 0$$

$$xye^{zx}\frac{\partial z}{\partial y} + e^{xy}\frac{\partial z}{\partial y} = -x^2e^{xy} - e^{zx} - xze^{xy}$$

$$\left[xye^{zx} + e^{xy}\right]\frac{\partial z}{\partial y} = -\left(x^2e^{xy} + e^{zx} + xze^{xy}\right)$$

$$\frac{\partial z}{\partial y} = -\frac{\left(x^2e^{xy} + e^{zx} + xze^{xy}\right)}{xye^{zx} + e^{xy}}$$

(23) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$$\begin{tikzcd} & z \arrow[dl, dash] \arrow[dr, dash] & \\ x & & y \end{tikzcd}$$

a, b, c are constants

Take $\frac{\partial}{\partial x}$: $\frac{2x}{a^2} + \frac{2z}{c^2} \frac{\partial z}{\partial x} = 0$

$$\frac{2z}{c^2} \frac{\partial z}{\partial x} = -\frac{2x}{a^2}$$

$$\frac{\partial z}{\partial x} = \frac{-2x}{a^2} \cdot \frac{c^2}{2z}$$

$$\frac{\partial z}{\partial x} = -\frac{c^2 x}{a^2 z}$$

Take $\frac{\partial}{\partial y}$: $\frac{2y}{b^2} + \frac{2z}{c^2} \frac{\partial z}{\partial y} = 0$

$$\frac{2z}{c^2} \frac{\partial z}{\partial y} = \frac{-2y}{b^2}$$

$$\frac{\partial z}{\partial y} = \frac{-2y}{b^2} \frac{c^2}{2z}$$

$$\frac{\partial z}{\partial y} = -\frac{c^2 y}{b^2 z}$$

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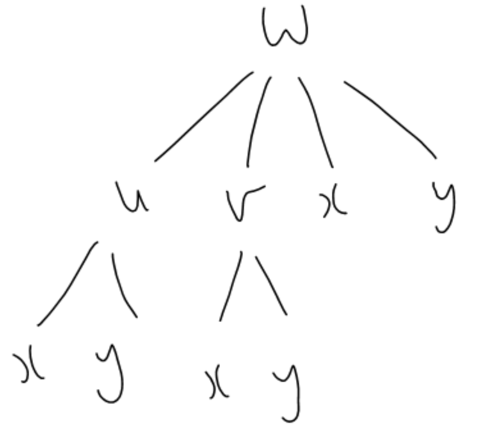


$\leftarrow w = f(u, v, x, y)$

Let $w = f(u, v, x, y)$

This is necessary because x appears on two different levels.

$$\frac{\partial w}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial f}{\partial x}$$

$$= 2u(1) + 2v(1) + 2x$$

$$= 2u + 2v + 2x$$

No need to simplify.

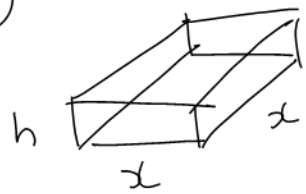
$$\frac{\partial w}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial f}{\partial y}$$

$$= 2u(-1) + 2v(1) + 2y$$

$$= -2u + 2v + 2y$$

No need to simplify.

(33)



(base is square)

$$V = x^2 h$$

$$\begin{tikzcd}
 & V \arrow[dl] \arrow[dr] & \\
x \arrow[d] & & h \arrow[d] \\
t & & t
\end{tikzcd}$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$= 2xh\frac{dx}{dt} + x^2\frac{dh}{dt}$$

Sub $h = 1\ ft$, $x = 2ft$, $\frac{dh}{dt} = -2\,\frac{in}{h} = \frac{-1}{6}\,\frac{ft}{h}$,

$$\frac{dx}{dt} = -3\,\frac{in}{h} = \frac{-1}{4}\,\frac{ft}{h}$$

$$\frac{dV}{dt} = 4\left(-\frac{1}{4}\right) + 4\left(-\frac{1}{6}\right)\ \frac{ft^3}{h}$$

$$= -\frac{5}{3}\ \frac{ft^3}{h}$$

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$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$= \frac{2}{3}\pi r h \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}$$

Sub $h = 5ft$, $r = 2ft$, $\frac{dh}{dt} = 0.4 \frac{ft}{min}$,

$\frac{dr}{dt} = 0.7 \frac{ft}{min}$

$$\frac{dV}{dt} = \frac{2}{3}\pi(10)(0.7) + \frac{1}{3}\pi(4)(0.4) \quad \frac{ft^3}{min}$$

$$= \frac{26\pi}{5} \quad \frac{ft^3}{min}$$

(37) $pV = nRT$

$$V = \frac{nRT}{p}$$

$$\begin{tikzcd}[column sep=small] & V \arrow[dl, no head] \arrow[dr, no head] & \\ T \arrow[d, no head] & & p \arrow[d, no head] \\ t & & t \end{tikzcd}$$

n and R are constants

$$\frac{dV}{dt} = \frac{\partial V}{\partial T}\frac{dT}{dt} + \frac{\partial V}{\partial p}\frac{dp}{dt}$$

$$= \frac{nR}{p}\frac{dT}{dt} - \frac{nRT}{p^2}\frac{dp}{dt}$$

Given $R = 82.06$, $p = 2\text{ atm}$, $T = 300\text{ k}$, $\frac{dp}{dt} = 1\ \frac{\text{atm}}{\text{min}}$, $\frac{dT}{dt} = 10\ \frac{\text{K}}{\text{min}}$

We can use $V = 10\text{L}$ and $pV = nRT$ to solve for n:

$$2(10) = n(82.06)(300)$$

$$n = \frac{20}{24618}$$

$$\frac{dV}{dt} = \frac{20}{24618}(82.06)\left(\frac{1}{2}\right)(10) - \frac{20}{24618}(82.06)(300)\left(\frac{1}{4}\right)(1)$$

$$\approx -5\ \frac{\text{L}}{\text{min}}$$ (Volume is decreasing)