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$$w = 3x^2 + 4xy - 2y^3$$

$$dw = w_x\,dx + w_y\,dy$$

$$= (6x + 4y)\,dx + (4x - 6y^2)\,dy$$

③ $w = \sqrt{1+x^2+y^2}$

$$w_x = \frac{1}{2}(1+x^2+y^2)^{-\frac{1}{2}}(2x)$$

$$= \frac{x}{\sqrt{1+x^2+y^2}}$$

$$w_y = \frac{1}{2}(1+x^2+y^2)^{-\frac{1}{2}}(2y)$$

$$= \frac{y}{\sqrt{1+x^2+y^2}}$$

$$dw = w_x\,dx + w_y\,dy$$

$$= \frac{x}{\sqrt{1+x^2+y^2}}dx + \frac{y}{\sqrt{1+x^2+y^2}}dy$$

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$$w = \arctan\left(\frac{x}{y}\right)$$

$$w_x = \frac{1}{1+\left(\frac{x}{y}\right)^2}\left(\frac{1}{y}\right)$$

$$w_y = \frac{1}{1+\left(\frac{x}{y}\right)^2}\left(-xy^{-2}\right)$$

$$= \frac{1}{1+\left(\frac{x}{y}\right)^2}\left(\frac{-x}{y^2}\right)$$

$$dw = w_x\,dx + w_y\,dy$$

$$= \frac{1}{1+\left(\frac{x}{y}\right)^2}\left(\frac{1}{y}\right)dx + \frac{1}{1+\left(\frac{x}{y}\right)^2}\left(\frac{-x}{y^2}\right)dy$$

No need to simplify.

9 $\quad w = x \tan yz$

$w_x = \tan yz$

$w_y = x \sec^2 yz \quad (z)$

$= xz \sec^2 yz$

$w_z = x \sec^2 yz \quad (y)$

$= xy \sec^2 yz$

$dw = w_x\,dx + w_y\,dy + w_z\,dz$

$= \tan yz\,dx + xz\sec^2 yz\,dy$

$+ xy \sec^2 yz\,dz$

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$$f = (1+x+y)^{-1}$$

$$f_x = -(1+x+y)^{-2}(1)$$

$$= \frac{-1}{(1+x+y)^2}$$

$$f_y = -(1+x+y)^{-2}(1)$$

$$= \frac{-1}{(1+x+y)^2}$$

$$df = f_x\,dx + f_y\,dy$$

$$= \frac{-1}{(1+x+y)^2}dx - \frac{1}{(1+x+y)^2}dy$$

$$f(x+dx,\ y+dy) \approx f(x,y) + df$$

$$\boxed{\text{Sub } x=3,\ dx=0.02,\ y=6,\ dy=0.05}$$

$$f(3.02, 6.05) \approx \frac{1}{10} - \frac{1}{100}(0.02) - \frac{1}{100}(0.05)$$

$$\approx 0.0993$$

(21) $f = \sqrt{x^2+y^2+z^2}$

$$f_x = \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}(2x)$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}}$$

Similarly, $f_y = \frac{y}{\sqrt{x^2+y^2+z^2}}$ and $f_z = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$$df = f_x\,dx + f_y\,dy + f_z\,dz$$

$$f(x+dx, y+dy, z+dz) \approx f(x,y,z) + df$$

$$f(x+dx, y+dy, z+dz) \approx \sqrt{x^2+y^2+z^2} + \frac{x}{\sqrt{x^2+y^2+z^2}}dx$$

$$+ \frac{y}{\sqrt{x^2+y^2+z^2}}dy + \frac{z}{\sqrt{x^2+y^2+z^2}}dz$$

$\boxed{\text{Sub } x=3,\ y=4,\ z=12,\ dx=0.03,\ dy=-0.04,\ dz=0.05}$

$$f(3.03, 3.96, 12.05) \approx 13 + \frac{3}{13}(0.03) + \frac{4}{13}(-0.04) + \frac{12}{13}(0.05)$$

$$\approx 13.04$$

(25) Let $f = (\sqrt{x} + \sqrt{y})^2$

Let $x = 16$ [because $\sqrt{16}$ is exact and $\sqrt{16} \approx \sqrt{15}$]

$y = 100$ [because $\sqrt{100}$ is exact and $\sqrt{100} \approx \sqrt{99}$]

Want $x + dx = 15$, $y + dy = 99 \implies dx = -1$, $dy = -1$

$$f_x = 2(\sqrt{x} + \sqrt{y}) \frac{1}{2} x^{-1/2}$$

$$= 1 + \frac{\sqrt{y}}{\sqrt{x}}$$

$$f_y = 2(\sqrt{x} + \sqrt{y}) \frac{1}{2} y^{-1/2}$$

$$= \frac{\sqrt{x}}{\sqrt{y}} + 1$$

$$df = f_x \, dx + f_y \, dy$$

$$f(x+dx, y+dy) \approx f(x,y) + df$$

$$\approx (\sqrt{x} + \sqrt{y})^2 + \left(1 + \frac{\sqrt{y}}{\sqrt{x}}\right) dx + \left(\frac{\sqrt{x}}{\sqrt{y}} + 1\right) dy$$

$\boxed{\text{Sub } x = 16,\ dx = -1,\ y = 100,\ dy = -1}$

$$f(15, 99) \approx 14^2 + \left(1 + \frac{10}{4}\right)(-1) + \left(\frac{4}{10} + 1\right)(-1)$$

$$\approx 191.1$$

(31) Curve: $2x^3+2y^3=9xy$

$$2x^3+2y^3-9xy=0$$

Let $f=2x^3+2y^3-9xy$

$$f_x=6x^2-9y$$
$$f_y=6y^2-9x$$
$$df=f_x\,dx+f_y\,dy$$
$$df=(6x^2-9y)\,dx+(6y^2-9x)\,dy$$

$$\boxed{\begin{array}{l}\text{Sub } x=1 \quad dx=0.1 \\ \quad\;\; y=2 \quad dy:\text{ unknown} \\ \text{Any point on the curve has } f=0. \\ \quad \text{Therefore } \Delta f=0 \\ \quad\quad \Rightarrow \; df\approx 0.\end{array}}$$

$$0\approx -12(0.1)+15\,dy$$
$$1.2\approx 15\,dy$$
$$dy\approx 0.08$$

Want $y+dy\approx 2.08$

(33) Area of a rectangle $A = bh$

Given $b = 10$ cm

$h = 15$ cm

$db_{max} = 0.1$ cm

$dh_{max} = 0.1$ cm

$$dA = A_b\, db + A_h\, dh$$
$$= h\, db + b\, dh$$

$$dA_{max} = h\, db_{max} + b\, dh_{max}$$
$$= 15(0.1) + 10(0.1)$$
$$= 2.5 \text{ cm}^2$$

$$\Delta A_{max} \approx 2.5 \text{ cm}^2$$

(37) $A = \frac{1}{2} ab\sin\theta$

$dA = A_a\, da + A_b\, db + A_\theta\, d\theta$

$= \frac{1}{2} b\sin\theta\, da + \frac{1}{2} a\sin\theta\, db + \frac{1}{2} ab\cos\theta\, d\theta$

To get dA_{max}, sub:

$a = 500$ ft $\qquad da_{max} = 1$ ft

$b = 700$ ft $\qquad db_{max} = 1$ ft

$\theta = 30^\circ \qquad d\theta_{max} = 0.25^\circ$

$= 0.25\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{720}$

$dA_{max} = \frac{1}{2}(700)\sin 30^\circ(1) + \frac{1}{2}(500)\sin 30^\circ(1)$
$+ \frac{1}{2}(500)(700)\cos 30^\circ\left(\frac{\pi}{720}\right)$

$\approx 961.2810 \text{ ft}^2$

$\approx 961.2810 \text{ ft}^2 \times \frac{1 \text{ acre}}{43560 \text{ ft}^2}$

≈ 0.02 acres

$\Delta A_{max} \approx 0.02$ acres

(39) $T = 2\pi \frac{\sqrt{L}}{\sqrt{g}}$

$$T_L = \frac{2\pi}{\sqrt{g}} \frac{1}{2} L^{-1/2} = \frac{\pi}{\sqrt{g}\sqrt{L}}$$

$$T_g = 2\pi\sqrt{L} \left[-\frac{1}{2} g^{-3/2} \right] = \frac{-\pi\sqrt{L}}{\sqrt{g}^3}$$

$$dT = T_L \, dL + T_g \, dg$$

$$= \frac{\pi}{\sqrt{g}\sqrt{L}} dL \quad \frac{-\pi\sqrt{L}}{\sqrt{g}^3} dg$$

Sub $L = 2$ ft $dL = \frac{1}{12}$ ft $\leftarrow$ Note: 1 inch $= \frac{1}{12}$ ft

$g = 32 \frac{ft}{s^2}$ $dg = 0.2 \frac{ft}{s^2}$

$$dT = \frac{\pi}{\sqrt{32}\sqrt{2}} \left(\frac{1}{12}\right) - \frac{\pi\sqrt{2}}{\sqrt{32}^3} (0.2)$$

≈ 0.03 seconds

$\Delta T \approx 0.03$ seconds

(41)

$$R = \frac{1}{32} v_0^2 \sin 2\alpha$$

$$R_{v_0} = \frac{1}{32}(2v_0)\sin 2\alpha$$
$$= \frac{1}{16} v_0 \sin 2\alpha$$

$$R_\alpha = \frac{1}{32} v_0^2 \, 2\cos 2\alpha$$
$$= \frac{1}{16} v_0^2 \cos 2\alpha$$

$$dR = R_{v_0}\, dv_0 + R_\alpha \, d\alpha$$
$$= \frac{1}{16} v_0 \sin 2\alpha \, dv_0 + \frac{1}{16} v_0^2 \cos 2\alpha \, d\alpha$$

$$\boxed{\begin{array}{ll} \text{Sub } v_0 = 400 \frac{ft}{s} & dv_0 = 10 \frac{ft}{s} \\ \alpha = 30^\circ & d\alpha = 1^\circ = \frac{\pi}{180} \end{array}}$$

$$dR = \frac{1}{16}(400)\sin 60^\circ (10) + \frac{1}{16}(400^2)\cos 60^\circ \left(\frac{\pi}{180}\right)$$
$$\approx 300 \text{ ft}$$

$$\Delta R \approx 300 \text{ ft}$$