⑤ $z = x^2 + y^2 - 6x + 2y + 5$

Set $z_x = 0$ and $z_y = 0$:

$$z_x = 2x - 6 = 0 \quad \Rightarrow \quad x = 3$$

$$z_y = 2y + 2 = 0 \quad \Rightarrow \quad y = -1$$

$$x = 3,\ y = -1 \rightarrow z = x^2 + y^2 - 6x + 2y + 5$$

$$= 9 + 1 - 18 - 2 + 5$$

$$= -5$$

$$(x, y, z) = (3, -1, -5)$$

(9) $z = 3x^2 + 12x + 4y^3 - 6y^2 + 5$

Set $z_x = 0$ and $z_y = 0$:

$z_x = 6x + 12 = 0 \Rightarrow x = -2$

$z_y = 12y^2 - 12y = 0 \Rightarrow 12y(y-1) = 0$

$y = 0$ or 1

The points have $x = -2$ and ($y = 0$ or $y = 1$).

Case 1: $x = -2, y = 0 \rightarrow z = 3x^2 + 12x + 4y^3 - 6y^2 + 5$

$= 12 - 24 + 5$

$= -7$

Case 2: $x = -2, y = 1 \rightarrow z = 3x^2 + 12x + 4y^3 - 6y^2 + 5$

$= 12 - 24 + 4 - 6 + 5$

$= -9$

The points are $(-2, 0, -7)$ and $(-2, 1, -9)$.

(11) $z = (2x^2+3y^2)e^{-x^2-y^2}$

$$z_x = 4x e^{-x^2-y^2} - 2x(2x^2+3y^2)e^{-x^2-y^2}$$
$$= 2x[2-2x^2-3y^2]e^{-x^2-y^2}$$

Set $z_x = 0$: $x=0$ or $2-2x^2-3y^2=0$

$$z_y = 6y e^{-x^2-y^2} - 2y(2x^2+3y^2)e^{-x^2-y^2}$$
$$= 2y[3-2x^2-3y^2]e^{-x^2-y^2}$$

Set $z_y=0$: $y=0$ or $3-2x^2-3y^2=0$

There are 4 cases: $x=0$ or $2-2x^2-3y^2=0$

AND

$y=0$ or $3-2x^2-3y^2=0$

Case 1: $x=0$ and $y=0$

$x=0,\ y=0 \rightarrow z = (2x^2+3y^2)e^{-x^2-y^2}$
$= 0$

Case 2: $x=0$ and $3-2x^2-3y^2=0$

$x=0 \rightarrow 3-2x^2-3y^2=0$
$3=3y^2$
$y^2=1$
$y=\pm 1$

Continued $\rightarrow$

(11) Cont'd

$x=0,\ y=\pm 1 \rightarrow z = (2x^2+3y^2)e^{-x^2-y^2}$

$= 3e^{-1}$

Case 3: $2-2x^2-3y^2=0$ and $y=0$

$y=0 \rightarrow 2-2x^2-3y^2=0$

$2-2x^2 = 0$

$2 = 2x^2$

$x^2 = 1$

$x = \pm 1$

$x=\pm 1,\ y=0 \rightarrow z = (2x^2+3y^2)e^{-x^2-y^2}$

$= 2e^{-1}$

Case 4: $2-2x^2-3y^2=0$ and $3-2x^2-3y^2=0$

$$\begin{cases} 2 = 2x^2+3y^2 \\ 3 = 2x^2+3y^2 \end{cases}$$

No Solution

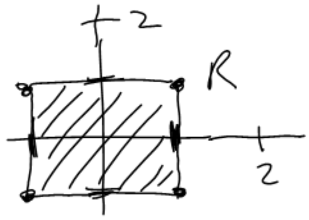
The points are

$(0,0,0)$

$(0,\pm 1, 3e^{-1})$

$(\pm 1, 0, 2e^{-1})$

(23) $f = x + 2y$



1) Interior Critical Points

$f_x = 1$

$f_y = 2$

Set $f_x = 0$ and $f_y = 0$: No Solution

No interior critical points.

2) Corners

$(-1,-1)$

$(-1,1)$

$(1,-1)$

$(1,1)$

3) Side 1 : $x = 1$

$f = 1 + 2y$

$f' = 2$

Set $f' = 0$: No Solution

No critical points on side 1

4) Side 2: $y = 1$

$f = x + 2$

$f' = 1$

Set $f' = 0$: No Solution

Continued →

(23) Cont'd

5) Side 3 : $x = -1$

$f = -1 + 2y$

$f' = 2$

Set $f' = 0$: No solution

6) Side 4 : $y = -1$

$f = x - 2$

$f' = 1$

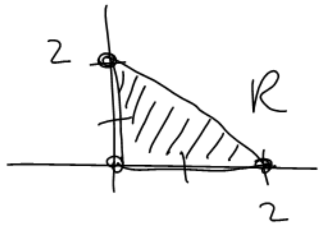
Set $f' = 0$: No Solution

Points	$f = x + 2y$	
$(-1, -1)$	-3	$\leftarrow$ absolute minimum
$(-1, 1)$	1	
$(1, -1)$	-1	
$(1, 1)$	3	$\leftarrow$ absolute maximum

The absolute minimum is $f = -3$, achieved at $(x, y) = (-1, -1)$.

The absolute maximum is $f = 3$, achieved at $(x, y) = (1, 1)$.

(25) $f = x^2 + y^2 - 2x$



1) Interior Critical Points

$f_x = 2x - 2 = 0 \Rightarrow x = 1$

$f_y = 2y = 0 \Rightarrow y = 0$

Note that $(1,0)$ is <u>not</u> in the interior of R.

No interior critical points.

2) Corners

$(0,0)$
$(2,0)$
$(0,2)$

3) Side 1: $x = 0$

$f = y^2$

$f' = 2y$

Set $f' = 0$: $2y = 0 \Rightarrow y = 0$

Note that $(0,0)$ is not on Side 1, it is a corner.

No critical points on Side 1.

4) Side 2: $y = 0$

$f = x^2 - 2x$

$f' = 2x - 2$

Set $f' = 0$: $x = 1$

$(1,0)$

Continued $\longrightarrow$

(25) Cont'd

5) Side 3: $y = mx + b$

$y = 2 - x$

$$f = x^2 + (2-x)^2 - 2x$$

$$f' = 2x + 2(2-x)(-1) - 2$$

$$= 2x - 2(2-x) - 2$$

$$= 2x - 4 + 2x - 2$$

$$= 4x - 6$$

Set $f' = 0$: $4x - 6 = 0$

$x = 3/2$

$x = 3/2 \rightarrow y = 2 - x$

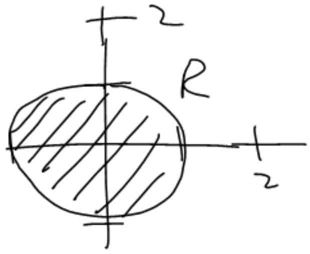
$y = \frac{1}{2}$

Points	$f = x^2 + y^2 - 2x$	
$(0,0)$	0	
$(2,0)$	0	
$(0,2)$	4	← absolute maximum
$(1,0)$	-1	← absolute minimum
$(\frac{3}{2}, \frac{1}{2})$	$-\frac{1}{2}$	

Absolute maximum of $f = 4$, achieved at $(x,y) = (0, 2)$.

Absolute minimum of $f = -1$, achieved at $(x,y) = (1, 0)$.

(27) $f = 2xy$



1) Interior Critical Points

$$f_x = 2y = 0 \Rightarrow y = 0$$
$$f_y = 2x = 0 \Rightarrow x = 0$$
$$(0,0)$$

A circle has 0 corners and 1 side.

2) Side: $x^2+y^2=1$

$$x^2+y^2=1$$
$$y^2 = 1-x^2$$
$$y = \pm\sqrt{1-x^2} \rightarrow f = 2xy$$
$$f = \pm 2x\sqrt{1-x^2}$$
$$f' = \pm\left[2x\,\tfrac{1}{2}(1-x^2)^{-1/2}(-2x) + 2\sqrt{1-x^2}\right]$$
$$= \pm\left[\frac{-2x^2}{\sqrt{1-x^2}} + 2\sqrt{1-x^2}\cdot\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\right]$$
$$= \pm\frac{2(1-x^2)-2x^2}{\sqrt{1-x^2}}$$
$$= \pm\frac{2-4x^2}{\sqrt{1-x^2}}$$
$$= \pm\frac{2(1-2x^2)}{\sqrt{1-x^2}}$$

Continued $\longrightarrow$

(27) Cont'd Critical Points: Points where $f' = 0$ or f' is undefined.

$f' = 0$: $1 - 2x^2 = 0$

$1 = 2x^2$

$\frac{1}{2} = x^2$

$x = \pm \frac{1}{\sqrt{2}}$

$x = \pm \frac{1}{\sqrt{2}} \rightarrow y^2 = 1 - x^2$

$y^2 = \frac{1}{2}$

$y = \pm \frac{1}{\sqrt{2}}$

f' is undefined: $1 - x^2 = 0$

$1 = x^2$

$x = \pm 1$

$x = \pm 1 \rightarrow y^2 = 1 - x^2$

$= 0$

$(\pm 1, 0)$

Points	$f = 2xy$
$(0,0)$	0
$(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$	1
$(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$	-1
$(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$	-1
$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$	1
$(1,0)$	0
$(-1,0)$	0

$\boxed{\text{Absolute minimum of } f = -1, \text{ achieved at } \pm(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}})}$

$\boxed{\text{Absolute maximum of } f = 1, \text{ achieved at } \pm(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})}$