(1) $$f = x^4 - x^3y + x^2y^2 - xy^3 + y^4$$

$$f_x = 4x^3 - 3x^2y + 2xy^2 - y^3$$

$$f_y = -x^3 + 2x^2y - 3xy^2 + 4y^3$$

⑤ Recall the Quotient Rule.

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

$$f = \frac{x+y}{x-y}$$

$$f_x = \frac{(x-y)(1) - (x+y)(1)}{(x-y)^2} = \frac{-2y}{(x-y)^2}$$

$$f_y = \frac{(x-y)(1) - (x+y)(-1)}{(x-y)^2} = \frac{2x}{(x-y)^2}$$

⑦ $f = \ln(x^2+y^2)$

$$f_x = \frac{1}{x^2+y^2}(2x) = \frac{2x}{x^2+y^2}$$

$$f_y = \frac{1}{x^2+y^2}(2y) = \frac{2y}{x^2+y^2}$$

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$$f = x^2 e^y \ln z$$

$$f_x = 2x e^y \ln z$$

$$f_y = x^2 e^y \ln z$$

$$f_z = x^2 e^y \left(\frac{1}{z}\right) = \frac{x^2 e^y}{z}$$

(19)

$$f = ue^v + ve^w + we^u$$

$$f_u = e^v + we^u$$

$$f_v = ue^v + e^w$$

$$f_w = ve^w + e^u$$

(21) $z = x^2 - 4xy + 3y^2$

$z_x = 2x - 4y$

$z_{xy} = -4$

$z_y = -4x + 6y$

$z_{yx} = -4$

$z_{xy} = z_{yx}$ ✓

23 Note: $\exp(-y^2)$ means e^{-y^2}

$z = x^2 e^{-y^2}$

$z_x = 2x e^{-y^2}$

$z_{xy} = 2x(-2y e^{-y^2}) = -4xy e^{-y^2}$

$z_y = x^2(-2y e^{-y^2}) = -2x^2 y e^{-y^2}$

$z_{yx} = -4xy e^{-y^2}$

$z_{xy} = z_{yx}$ ✓

(27) $z = e^{-3x} \cos y$

$z_x = -3e^{-3x} \cos y$

$z_{xy} = -3e^{-3x}(-\sin y) = 3e^{-3x} \sin y$

$z_y = e^{-3x}(-\sin y) = -e^{-3x} \sin y$

$z_{yx} = 3e^{-3x} \sin y$

$z_{xy} = z_{yx}$

(31) $z = x^2 + y^2 \qquad P = (3, 4, 25)$

$z_x = 2x = 6$

$z_y = 2y = 8$

$\vec{n} = [-z_x, -z_y, 1]$

$= [-6, -8, 1]$

$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

$$\begin{bmatrix} -6 \\ -8 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ -8 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \\ 25 \end{bmatrix}$$

$-6x - 8y + z = -25$

(35) $z = x^3 - y^3 \qquad P = (3, 2, 19)$

$$z_x = 3x^2 = 27$$

$$z_y = -3y^2 = -12$$

$$\vec{n} = [-z_x, -z_y, 1]$$

$$= [-27, 12, 1]$$

$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$$

$$\begin{bmatrix} -27 \\ 12 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -27 \\ 12 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \\ 19 \end{bmatrix}$$

$$-27x + 12y + z = -38$$

37 $z = xy \quad P = (1, -1, -1)$

$z_x = y = -1$

$z_y = x = 1$

$\vec{n} = [-z_x, -z_y, 1]$

$= [1, -1, 1]$

$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{P}$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$x - y + z = 1$

(53) $f = e^{xyz}$

$f_x = yze^{xyz}$ $\quad f_y = xze^{xyz}$ $\quad f_z = xye^{xyz}$

$f_{xx} = y^2z^2e^{xyz}$

$f_{xy} = ze^{xyz} + yz(xze^{xyz})$

$= ze^{xyz} + xyz^2e^{xyz}$

$f_{xz} = ye^{xyz} + yz(xye^{xyz})$

$= ye^{xyz} + xy^2ze^{xyz}$

$f_{yy} = x^2z^2e^{xyz}$

$f_{yx} = f_{xy}$

$f_{yz} = xe^{xyz} + xz(xye^{xyz})$

$= xe^{xyz} + x^2yze^{xyz}$

$f_{zz} = x^2y^2e^{xyz}$

$f_{zx} = f_{xz}$

$f_{zy} = f_{yz}$

Continued $\longrightarrow$

53 Cont'd

From above, $f_{xy} = ze^{xyz} + xyz^2e^{xyz}$

$$f_{xyz} = e^{xyz} + z(xye^{xyz}) + 2xyze^{xyz} + xyz^2(xye^{xyz})$$

$$= e^{xyz} + 3xyze^{xyz} + x^2y^2z^2e^{xyz}$$

$$= (1+3xyz+x^2y^2z^2)e^{xyz}$$

(55) Note: $\exp(-n^2kt)$ means e^{-n^2kt}

Given $u = e^{-n^2kt} \sin nx$,

show that $u_t = k u_{xx}$.

$$u_t = -n^2 k e^{-n^2kt} \sin nx$$

$$u_x = n e^{-n^2kt} \cos nx$$

$$u_{xx} = -n^2 e^{-n^2kt} \sin nx$$

$$LS = u_t$$
$$= -n^2 k e^{-n^2kt} \sin nx$$

$$RS = k u_{xx}$$
$$= -n^2 k e^{-n^2kt} \sin nx$$

$LS = RS$ ✓

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$$pV = nRT$$

$$p = \frac{nRT}{V}$$

$$p_V = \frac{-nRT}{V^2}$$

$$V = \frac{nRT}{p}$$

$$V_T = \frac{nR}{p}$$

$$T = \frac{pV}{nR}$$

$$T_p = \frac{V}{nR}$$

$$LS = p_V \cdot V_T \cdot T_p$$

$$= \frac{-nRT}{V^2} \cdot \frac{nR}{p} \cdot \frac{V}{nR}$$

$$= \frac{-nRT}{pV}$$

$$\boxed{\text{But } pV = nRT}$$

$$= \frac{-nRT}{nRT}$$

$$= -1$$

$$= RS$$

(65) $z = x^2 + 2xy + 2y^2 - 6x + 8y$

$z_x = 2x + 2y - 6$

$z_y = 2x + 4y + 8$

Set $z_x = 0$ and $z_y = 0$:

$2x + 2y - 6 = 0 \Rightarrow 2x + 2y = 6$

$2x + 4y + 8 = 0 \Rightarrow 2x + 4y = -8$

Solve the system using any method.

Cramer's Rule:

$$x = \frac{\begin{vmatrix} 6 & 2 \\ -8 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 2 \\ 2 & 4 \end{vmatrix}} = \frac{40}{4} = 10$$

$$y = \frac{\begin{vmatrix} 2 & 6 \\ 2 & -8 \end{vmatrix}}{\begin{vmatrix} 2 & 2 \\ 2 & 4 \end{vmatrix}} = \frac{-28}{4} = -7$$

$x = 10,\ y = -7 \rightarrow z = x^2 + 2xy + 2y^2 - 6x + 8y$

$= 100 - 140 + 98 - 60 - 56$

$= -58$

$(x, y, z) = (10, -7, -58)$