(5) $f = 2x^2 + 2xy + y^2 + 4x - 2y + 1$

$f_x = 4x + 2y + 4$

$f_y = 2x + 2y - 2$

$4x + 2y + 4 = 0$ (1)

$2x + 2y - 2 = 0$ (2)

(1) - (2): $2x + 6 = 0$

$x = -3$

$x = -3 \rightarrow$ (1): $-12 + 2y + 4 = 0$

$2y = 8$

$y = 4$

The point is $(-3, 4)$

$f_{xx} = 4$, $f_{yy} = 2$, $f_{xy} = 2$

Point	f_{xx}	f_{yy}	f_{xy}	Δ	Conclude
$(-3, 4)$	4	2	2	> 0	Local Min

(7) $f = x^3 + y^3 + 3xy + 3$

$f_x = 3x^2 + 3y$

$f_y = 3y^2 + 3x$

$3x^2 + 3y = 0$ (1)

$3y^2 + 3x = 0$ (2)

(1): $3y = -3x^2$

$y = -x^2$

$y = -x^2 \rightarrow$ (2):

$3x^4 + 3x = 0$

$3x(x^3 + 1) = 0$

$x = 0, \quad x^3 + 1 = 0$

$x^3 = -1$

$x = -1$

$x = 0, x = -1$ together with $y = -x^2$ gives points $(0,0)$ and $(-1,-1)$.

$f_{xx} = 6x, \quad f_{yy} = 6y, \quad f_{xy} = 3$

Point	f_{xx}	f_{yy}	f_{xy}	Δ	Conclude
$(0,0)$	0	0	3	<0	Saddle Point
$(-1,-1)$	-6	-6	3	>0	Local Max

(11) $f = x^4 + y^4 - 4xy$

$f_x = 4x^3 - 4y$

$f_y = 4y^3 - 4x$

$4x^3 - 4y = 0$ (1)

$4y^3 - 4x = 0$ (2)

(1): $4x^3 = 4y$

$y = x^3$

$y = x^3 \rightarrow$ (2): $4x^9 - 4x = 0$

$4x(x^8 - 1) = 0$

$4x(x^4 - 1)(x^4 + 1) = 0$

$4x(x^2 - 1)(x^2 + 1)(x^4 + 1) = 0$

$4x(x-1)(x+1)(x^2+1)(x^4+1) = 0$

$x = 0, \pm 1$

no solution to $x^2 + 1 = 0$, $x^4 + 1 = 0$

$x = 0, \pm 1$ together with $y = x^3$ gives points $(0,0)$, $(-1,-1)$, $(1,1)$.

$f_{xx} = 12x^2$

$f_{xy} = -4$

$f_{yy} = 12y^2$

Continued →

(11) Cont'd

Point	f_{xx}	f_{yy}	f_{xy}	Δ	Conclude
$(0,0)$	0	0	-4	<0	Saddle Point
$(-1,-1)$	12	12	-4	>0	Local Min
$(1,1)$	12	12	-4	>0	Local Min

(13) $f = x^3 + 6xy + 3y^2 - 9x$

$f_x = 3x^2 + 6y - 9$

$f_y = 6x + 6y$

$3x^2 + 6y - 9 = 0$ (1)

$6x + 6y = 0$ (2)

(2): $6x + 6y = 0$

$6y = -6x$

$y = -x$

$y = -x \Rightarrow$ (1): $3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$x = -3, -1$

$x = 3, -1$ together with $y = -x$ gives points $(3, -3)$ and $(-1, 1)$.

$f_{xx} = 6x$

$f_{yy} = 6$

$f_{xy} = 6$

Point	f_{xx}	f_{yy}	f_{xy}	Δ	Conclude
$(3,-3)$	18	6	6	>0	Local Min
$(-1,1)$	-6	6	6	<0	Saddle Point

(15) $f = 3x^2 + 6xy + 2y^3 + 12x - 24y$

$f_x = 6x + 6y + 12$

$f_y = 6x + 6y^2 - 24$

$6x + 6y + 12 = 0$ (1)

$6x + 6y^2 - 24 = 0$ (2)

(1) $6y = -6x - 12$

$y = -x - 2$

$y = -x - 2 \rightarrow$ (2): $6x + 6(-x-2)^2 - 24 = 0$

$6x + 6(x^2 + 4x + 4) - 24 = 0$

$6x + 6x^2 + 24x + 24 - 24 = 0$

$6x^2 + 30x = 0$

$6x(x+5) = 0$

$x = 0, -5$

$x = 0, -5$ together with $y = -x - 2$ gives points $(0, -2)$ and $(-5, 3)$.

$f_{xx} = 6$

$f_{xy} = 6$

$f_{yy} = 12y$

Point	f_{xx}	f_{yy}	f_{xy}	Δ	Conclude
$(0, -2)$	6	-24	6	<0	Saddle Point
$(-5, 3)$	6	36	6	>0	Local Min

(17) $f = 4xy - 2x^4 - y^2$

$f_x = 4y - 8x^3$

$f_y = 4x - 2y$

$4y - 8x^3 = 0$ (1)

$4x - 2y = 0$ (2)

(2): $4x - 2y = 0$

$4x = 2y$

$y = 2x$

$y = 2x \rightarrow$ (1): $8x - 8x^3 = 0$

$8x(1 - x^2) = 0$

$8x(1-x)(1+x) = 0$

$x = 0, \pm 1$

$x = 0, \pm 1$ together with $y = 2x$ gives points $(0,0)$, $(-1,-2)$, $(1,2)$.

$f_{xx} = -24x^2$

$f_{yy} = -2$

$f_{xy} = 4$

Point	f_{xx}	f_{yy}	f_{xy}	Δ	Conclude
$(0,0)$	0	-2	4	<0	Saddle Point
$(-1,-2)$	-24	-2	4	>0	Local Max
$(1,2)$	-24	-2	4	>0	Local Max

(19) $f = 2x^3 - 3x^2 + y^2 - 12x + 10$

$f_x = 6x^2 - 6x - 12$

$f_y = 2y$

$6x^2 - 6x - 12 = 0$ (1)

$6(x^2 - x - 2) = 0$

$6(x-2)(x+1) = 0$

$x = -1, 2$

$2y = 0$ (2)

$y = 0$

The points are $(-1,0)$ and $(2,0)$.

$f_{xx} = 12x - 6$

$f_{yy} = 2$

$f_{xy} = 0$

Point	f_{xx}	f_{yy}	f_{xy}	Δ	Conclude
$(-1,0)$	-18	2	0	<0	Saddle Point
$(2,0)$	18	2	0	>0	Local Min