8.1 Integrating Trig Functions

$$\int \sin x\,dx = -\cos x + C$$
$$\int \cos x\,dx = \sin x + C$$
$$\int \tan x\,dx = \ln|\sec x| + C$$
$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$
$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$
$$\int \cot x\,dx = -\ln|\csc x| + C$$

$$\int \sec^2 x\,dx = \tan x + C$$
$$\int \sec x \tan x\,dx = \sec x + C$$
$$\int \csc x \cot x\,dx = -\csc x + C$$
$$\int \csc^2 x\,dx = -\cot x + C$$

Ex: Find $\int \frac{1}{\sqrt{x}\sin\sqrt{x}}\,dx$

$$= \int \frac{\csc\sqrt{x}}{\sqrt{x}}\,dx$$

$u = \sqrt{x}$
$du = \frac{1}{2}x^{-1/2}\,dx$
$2du = \frac{dx}{\sqrt{x}}$

$$= 2\int \csc u\,du$$
$$= -2\ln|\csc u + \cot u| + C$$
$$= -2\ln|\csc\sqrt{x} + \cot\sqrt{x}| + C$$

Ex: $\int \frac{e^x \cos e^x}{\sin e^x}\, dx$

$= \int e^x \cot e^x \, dx$

$u = e^x$
$du = e^x dx$

$= \int \cot u \, du$

$= -\ln|\csc u| + C$

$= -\ln|\csc e^x| + C$

Ex: $\int \frac{\sec(\ln x)\tan(\ln x)}{x}\, dx$

$= \int \sec u \tan u \, du$

$u = \ln x$
$du = \frac{1}{x} dx$

$= \sec u + C$

$= \sec(\ln x) + C$

Recall

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

Mixed Practice

Ex: Find $\int \frac{(1+\ln x)^3}{x}\,dx$

$u = 1 + \ln x$

$du = \frac{1}{x}\,dx$

$$= \int u^3\,du$$

$$= \frac{u^4}{4} + C$$

$$= \frac{1}{4}(1+\ln x)^4 + C$$

Ex: Find:

a) $\int \frac{x}{9+x^2}\,dx$

$u = 9 + x^2$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$= \frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|9+x^2| + C$

b) $\int \frac{x}{9+x^4}\,dx$

$= \int \frac{x}{9+(x^2)^2}\,dx$

$u = x^2$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$= \frac{1}{2}\int \frac{du}{3^2+u^2}$

$= \frac{1}{6}\arctan\frac{u}{3} + C$

$= \frac{1}{6}\arctan\frac{x^2}{3} + C$

Ex: Find $\int (\sec^2 x) e^{\tan x} \, dx$

$u = \tan x$
$du = \sec^2 x \, dx$

$$= \int e^u \, du$$
$$= e^u + C$$
$$= e^{\tan x} + C$$

Ex: Find $\int \frac{e^{4x}}{\sqrt{9 - e^{8x}}} \, dx$

$$= \int \frac{e^{4x}}{\sqrt{3^2 - (e^{4x})^2}} \, dx$$

$u = e^{4x}$
$du = 4e^{4x} \, dx$
$\frac{du}{4} = e^{4x} \, dx$

$$= \frac{1}{4} \int \frac{du}{\sqrt{3^2 - u^2}}$$
$$= \frac{1}{4} \arcsin \frac{u}{3} + C$$
$$= \frac{1}{4} \arcsin \frac{e^{4x}}{3} + C$$