# 5.9 Hyperbolic Functions

Hyperbolic Cosine $\cosh x = \frac{e^x + e^{-x}}{2}$

" Sine $\sinh x = \frac{e^x - e^{-x}}{2}$

Quick Ex:

Hanging cable

$y = a\cosh\left(\frac{x}{a}\right)$, $a > 0$

The shape is called a catenary.

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Less commonly-used:

$\tanh x = \frac{\sinh x}{\cosh x}$ $\quad \coth x = \frac{1}{\tanh x}$ $\quad \operatorname{csch} x = \frac{1}{\sinh x}$

$\operatorname{sech} x = \frac{1}{\cosh x}$

$(\cosh t, \sinh t)$ lies on the hyperbola $x^2 - y^2 = 1$

Hence the name "hyperbolic functions"

## Properties

$\cos^2\theta + \sin^2\theta = 1$

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$\cosh^2 x - \sinh^2 x = 1$

$\cosh(-x) = \cosh x$

$\sinh(-x) = -\sinh x$

$$\frac{d}{dx}[\cosh x] = \sinh x \qquad \frac{d}{dx}[\sinh x] = \cosh x$$

$$\int \cosh x\, dx = \sinh x + C \qquad \int \sinh x\, dx = \cosh x + C$$

**Ex:** Find $y'$

a) $y = \sinh x^3$

$y' = 3x^2 \cosh x^3$

b) $y = x^2 \cosh(5x)$

$y' = 5x^2 \sinh(5x) + 2x \cosh(5x)$

**Ex:** Find:

a) $\int \cosh(4x)\, dx$

$= \frac{1}{4}\sinh(4x) + C$

b) $\int x^6 \sinh x^7\, dx$

$= \frac{1}{7}\cosh x^7 + C$