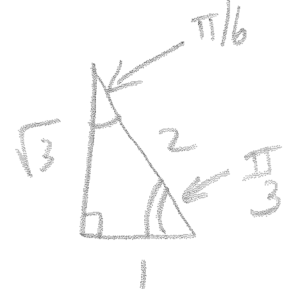
5.7 Derivatives of Inverse Trig Functions

$\sin(\text{angle}) = \#$

$\arcsin(\#) = \text{angle}$

$\arcsin x = \sin^{-1} x$

Not to be confused with $(\sin x)^{-1} = \frac{1}{\sin x} = \csc x$

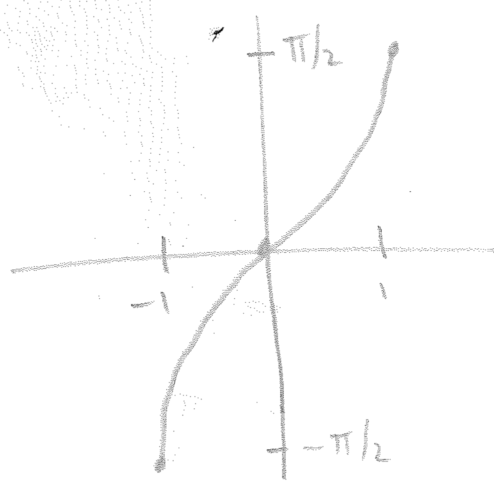


$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

$\sin\frac{\pi}{6} = \frac{1}{2}$ $\quad \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$$\boxed{\sin^{-1}(-a) = -\sin^{-1} a}$$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$



$y = \sin^{-1} x$

$-1 \leq x \leq 1$

$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$

$$\tan(\text{angle}) = \#$$

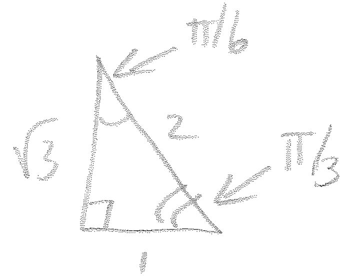
$$\arctan(\#) = \text{angle}$$

$$\arctan x = \tan^{-1} x$$

Not to be confused with $(\tan x)^{-1} = \frac{1}{\tan x} = \cot x$

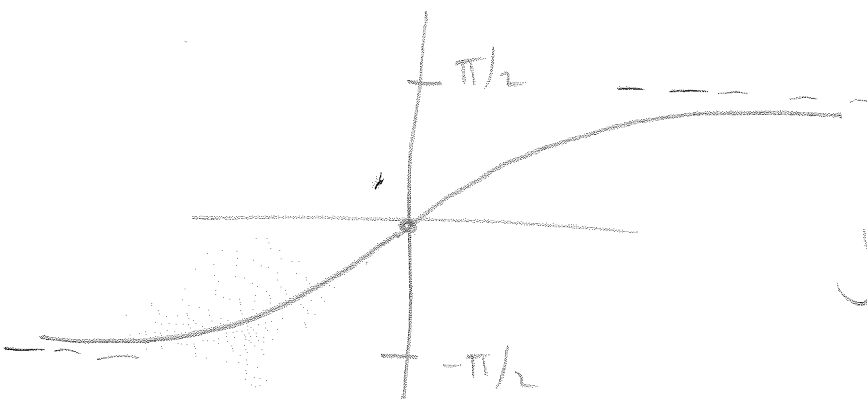
$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$ $\qquad \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$\tan \frac{\pi}{3} = \sqrt{3}$ $\qquad \tan^{-1} \sqrt{3} = \frac{\pi}{3}$



$$\boxed{\arctan(-a) = -\arctan a}$$

$$\arctan(-1) = -\frac{\pi}{4}$$



$y = \tan^{-1} x$

$$-\infty < x < \infty$$

$$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$$

Ex: Solve for x

$$\arctan(2x-1) = \frac{\pi}{3}$$

$$\tan(\arctan(2x-1)) = \tan\frac{\pi}{3}$$

$$2x-1 = \sqrt{3}$$

$$2x = 1+\sqrt{3}$$

$$x = \frac{1+\sqrt{3}}{2}$$

$$\boxed{\begin{aligned} \frac{d}{dx}\sin^{-1}u &= \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \\ \frac{d}{dx}\tan^{-1}u &= \frac{1}{1+u^2}\frac{du}{dx} \end{aligned}}$$

Ex: Find y'

a) $y = \sin^{-1} 5x$

$$y' = \frac{1}{\sqrt{1-(5x)^2}} \cdot 5$$

$$= \frac{5}{\sqrt{1-25x^2}}$$

b) $y = \tan^{-1} 4x^5$

$$y' = \frac{1}{1+(4x^5)^2} \cdot 20x^4$$

$$= \frac{20x^4}{1+16x^{10}}$$

c) $y = 2x \arctan x - \ln(1+x^2)$

$$y' = 2x \cdot \frac{1}{1+x^2} + 2\arctan x - \frac{2x}{1+x^2}$$

$$= 2\arctan x$$

d) $y = 12 \arcsin \frac{x}{4}$

$$y' = \frac{12}{\sqrt{1-\left(\frac{x}{4}\right)^2}} \cdot \frac{1}{4}$$

$$= \frac{12}{\sqrt{1-\frac{x^2}{16}}\ \sqrt{16}}$$

$$= \frac{12}{\sqrt{16-x^2}}$$

Ex: Prove that $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$

$$\sin(\arcsin x) = x$$

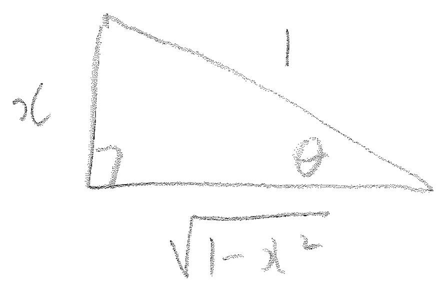
Take $\frac{d}{dx}$: $\cos(\arcsin x) \frac{d}{dx} \arcsin x = 1$

$$\frac{d}{dx} \arcsin x = \frac{1}{\cos(\arcsin x)}$$

Let $\theta = \arcsin x$

$\sin\theta = x$

$\sin\theta = \frac{x}{1}$



$\cos\theta = \sqrt{1-x^2}$

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$