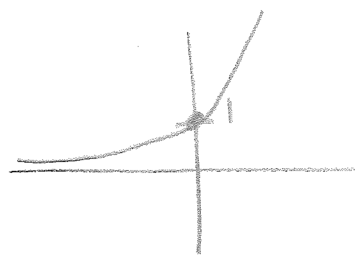
5.1 Derivatives of Exponentials and Logs

Exponential Function:

$f(x) = b^x \quad b:$ Constant

The most important base is $e \approx 2.718$



$y = e^x$

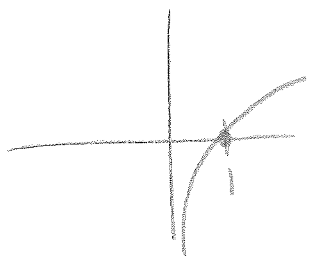
$$\boxed{\begin{aligned} \frac{d}{dx} e^x &= e^x \\ \frac{d}{dx} e^u &= e^u \frac{du}{dx} \end{aligned}}$$

$$\boxed{\begin{aligned} e^a e^b &= e^{a+b} \\ \frac{e^a}{e^b} &= e^{a-b} \\ (e^a)^b &= e^{ab} \end{aligned}}$$

Logarithmic Function:

$f(x) = \log_b x$ $\quad$ b: Constant

Note: $\log_e x$ is written $\ln x$



Ex: Find $\lim_{x \to 7^+} \ln(x-7)$

$= \ln 0^+$

$= -\infty$

$$\boxed{\begin{aligned} \frac{d}{dx} \ln x &= \frac{1}{x} \\ \frac{d}{dx} \ln u &= \frac{1}{u}\left(\frac{du}{dx}\right) \end{aligned}}$$

$$\boxed{\begin{aligned} \ln(ab) &= \ln a + \ln b \\ \ln\left(\frac{a}{b}\right) &= \ln a - \ln b \\ \ln a^b &= b \ln a \end{aligned}}$$

Ex: Find y'

a) $y = e^{3x}$

$y' = 3e^{3x}$

b) $y = \ln(x^3+1)$

$y' = \frac{1}{x^3+1} \cdot 3x^2$

$= \frac{3x^2}{x^3+1}$

c) $y = e^{-2x} \sin 5x$

$y' = e^{-2x}(5\cos 5x) + \sin 5x(-2e^{-2x})$

$= e^{-2x}(5\cos 5x - 2\sin 5x)$

d) $y = \ln \sqrt{\frac{x+1}{2x+3}}$

$= \ln \left(\frac{x+1}{2x+3}\right)^{1/2}$

$= \frac{1}{2} \ln \left(\frac{x+1}{2x+3}\right)$

$= \frac{1}{2}\left[\ln(x+1) - \ln(2x+3)\right]$

$y' = \frac{1}{2}\left[\frac{1}{x+1} - \frac{2}{2x+3}\right]$

e) $y = e^{x^2} + \cos(\ln x)$

$y' = 2xe^{x^2} - \sin(\ln x) \cdot \frac{1}{x}$

$= 2xe^{x^2} - \frac{\sin(\ln x)}{x}$

Ex: Let $y = e^{2x} - 5x$

Find x so that $y' = 0$.

$y' = 2e^{2x} - 5$

$0 = 2e^{2x} - 5$

$5 = 2e^{2x}$

$\frac{5}{2} = e^{2x}$

$\ln \frac{5}{2} = 2x$

$x = \frac{1}{2} \ln \frac{5}{2}$