4.4–4.5 Review of Integration

The indefinite integral $\int f(x)\,dx$ represents all antiderivatives of $f(x)$.

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad (n \neq -1)}$$

Ex: Find:

a) $\int (x^5 + 2x^4 - 5x + 3)\,dx$

$= \frac{x^6}{6} + \frac{2x^5}{5} - \frac{5x^2}{2} + 3x + C$

b) $\int \left(\sqrt{x} + \frac{1}{x^3}\right)dx$

$= \int \left(x^{1/2} + x^{-3}\right)dx$

$= \frac{2}{3}x^{3/2} - \frac{1}{2}x^{-2} + C$

Ex: Find:

a) $\int 5x^2 (x^3+1)^6 \, dx$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2 \, dx$

$\frac{du}{3} = x^2 \, dx$

$= \frac{5}{3} \int u^6 \, du$

$= \frac{5}{3} \frac{u^7}{7} + C$

$= \frac{5}{21} (x^3+1)^7 + C$

b) $\int \frac{x}{\sqrt{2x^2+1}} \, dx$

$u = 2x^2+1$

$du = 4x \, dx$

$\frac{du}{4} = x \, dx$

$= \frac{1}{4} \int \frac{du}{\sqrt{u}}$

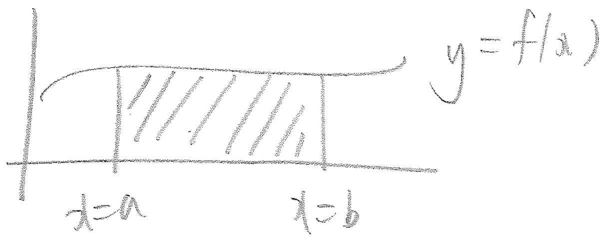
$= \frac{1}{4} \int u^{-1/2} \, du$

$= \frac{1}{4} (2u^{1/2}) + C$

$= \frac{1}{2} \sqrt{2x^2+1} + C$

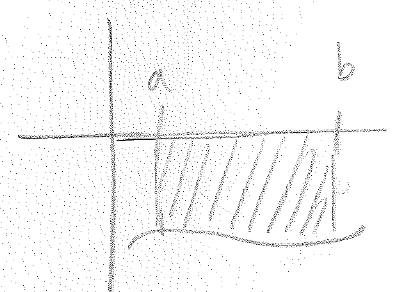
The definite integral $\int_a^b f(x)\,dx$

represents area under a curve, if $f(x) \geq 0$.



$$\int_a^b f(x)\,dx = F(b) - F(a), \text{ where } F(x) \text{ is an antiderivative of } f(x)$$

$$= F(x)\Big|_a^b$$



$\int_a^b f(x)\,dx < 0$

when the curve is below the x-axis

Ex: Evaluate $\int_1^2 (x^2-4)\,dx$

$$= \left[\frac{x^3}{3} - 4x\right]_1^2$$

$$= \left(\frac{8}{3} - 8\right) - \left(\frac{1}{3} - 4\right)$$

$$= \frac{7}{3} - 4$$

$$= -\frac{5}{3}$$

Ex: Evaluate $\int_0^2 2x(x^2+1)^3\,dx$

Method 1

$$\boxed{\begin{aligned} u &= x^2+1 \\ du &= 2x\,dx \end{aligned}}$$

$$= \int_{x=0}^{x=2} u^3\,du$$

$$= \frac{u^4}{4}\Big|_{x=0}^{x=2}$$

$$= \frac{1}{4}(x^2+1)^4 \Big|_0^2$$

$$= \frac{5^4}{4} - \frac{1}{4}$$

$$= \frac{624}{4}$$

$$= 156$$

Method 2

$$\int_0^2 2x(x^2+1)^3\,dx$$

$$\boxed{\begin{gathered} u = x^2+1 \\ du = 2x\,dx \\ x=0 \Rightarrow u=1 \\ x=2 \Rightarrow u=5 \end{gathered}}$$

$$= \int_1^5 u^3\,du$$

$$= \left.\frac{u^4}{4}\right|_1^5$$

$$= \frac{5^4}{4} - \frac{1}{4}$$

$$= 156$$

Ex: Evaluate $\displaystyle\int_0^1 \frac{x}{(x^2+1)^4}\,dx$

$$\boxed{\begin{gathered} u = x^2+1 \\ du = 2x\,dx \\ \frac{du}{2} = x\,dx \\ x=0 \Rightarrow u=1 \\ x=1 \Rightarrow u=2 \end{gathered}}$$

$$= \frac{1}{2}\int_1^2 \frac{du}{u^4}$$

$$= \frac{1}{2}\int_1^2 u^{-4}\,du$$

$$= \left. -\frac{1}{6}u^{-3} \right|_1^2$$

$$= -\frac{1}{6}\left(\frac{1}{8}-1\right)$$

$$= \frac{7}{48}$$