2.5 Implicit Differentiation

y is an <u>explicit</u> function of x: $y = \pm\sqrt{25 - x^2}$

" <u>implicit</u> " $x^2 + y^2 = 25$

<u>Ex</u>: y depends on x

Find

a) $\frac{d}{dx}[x^2]$

$= 2x$

b) $\frac{d}{dx}[y^2]$

$= 2y\frac{dy}{dx}$ Chain Rule

c) $\frac{d}{dx}[(4x^2)y^3]$

$= 4x^2(3y^2\frac{dy}{dx}) + y^3(8x)$

$= 12x^2y^2\frac{dy}{dx} + 8xy^3$

<u>Ex</u>: Find $\frac{dy}{dx}$ given $x^2 + y^2 = 25$

1) Take $\frac{d}{dx}$

$2x + 2y\frac{dy}{dx} = 0$

2) Solve for $\frac{dy}{dx}$

$\frac{dy}{dx} = -\frac{2x}{2y}$ or $-\frac{x}{y}$

Ex: Find the tangent line to $x^3 + y^3 = 9xy$ at $(2,4)$

$(9x)y$

Need $\frac{dy}{dx}$

1) Take $\frac{d}{dx}$

$$3x^2 + 3y^2 \frac{dy}{dx} = 9x \frac{dy}{dx} + y(9)$$

2) Solve for $\frac{dy}{dx}$

$$3y^2 \frac{dy}{dx} - 9x \frac{dy}{dx} = 9y - 3x^2$$

$$[3y^2 - 9x] \frac{dy}{dx} = 9y - 3x^2$$

$$\frac{dy}{dx} = \frac{9y - 3x^2}{3y^2 - 9x}$$

$$\left.\frac{dy}{dx}\right|_{(2,4)} = \frac{24}{30} \text{ or } \frac{4}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{4}{5}(x - 2)$$

$$y - 4 = \frac{4}{5}x - \frac{8}{5}$$

$$y = \frac{4}{5}x - \frac{8}{5} + 4$$

$$y = \frac{4}{5}x + \frac{12}{5}$$

Ex: Find $\frac{dy}{dx}$ given $x^4y + y^4 = 1 + \sin(xy)$

1) Take $\frac{d}{dx}$:

$$x^4\frac{dy}{dx} + y(4x^3) + 4y^3\frac{dy}{dx} = \cos(xy)\left[x\frac{dy}{dx} + y(1)\right]$$

$$x^4\frac{dy}{dx} + 4x^3y + 4y^3\frac{dy}{dx} = x\cos(xy)\frac{dy}{dx} + y\cos xy$$

$$\left[x^4 + 4y^3 - x\cos xy\right]\frac{dy}{dx} = y\cos xy - 4x^3y$$

$$\frac{dy}{dx} = \frac{y\cos xy - 4x^3y}{x^4 + 4y^3 - x\cos xy}$$