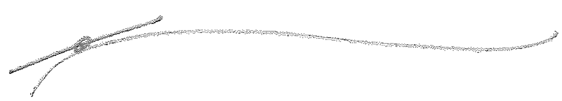
# 2.2–2.4 Review of Derivatives: Derivative Rules and Trig

The derivative of $y=f(x)$ can be written:

$y'$; $f'(x)$, $\frac{dy}{dx}$ or $\frac{df}{dx}$.

Evaluating: $y'\big|_{x=a}$, $f'(a)$, $\frac{dy}{dx}\Big|_{x=a}$, $\frac{df}{dx}\Big|_{x=a}$.

$f'(x)$ represents: slope of tangent line to $y=f(x)$
<u>and</u> instantaneous rate of change of $f$ with respect to $x$




Power Rule $\quad \frac{d}{dx}[x^n] = nx^{n-1}$

Product Rule $\quad [fg]' = fg' + gf'$

Quotient Rule $\quad \left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2}$

Chain Rule: Calculation Version $\quad [f(g(x))]' = f'(g(x)) \cdot g'(x)$

Chain Rule: Formal Version $\quad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Ex: $y = \sqrt[3]{x^3+1}$ Find $y'$

$y = (x^3+1)^{1/3}$

$y' = \frac{1}{3}(x^3+1)^{-2/3} \cdot 3x^2$

$= \dfrac{x^2}{\sqrt[3]{x^3+1}^{\,2}}$

Ex: Confirm using the formal Chain Rule

$y = \sqrt[3]{u} \qquad u = x^3+1$

$\dfrac{dy}{dx} = \dfrac{dy}{du}\,\dfrac{du}{dx}$

$= \frac{1}{3}u^{-2/3}(3x^2)$

$= \dfrac{x^2}{\sqrt[3]{x^3+1}^{\,2}}$

Ex: Find $\frac{dy}{dx}$

a) $y = 5x^4 + \frac{3}{x^2} + 6\sqrt{x}$

$y = 5x^4 + 3x^{-2} + 6x^{1/2}$

$\frac{dy}{dx} = 20x^3 - 6x^{-3} + 3x^{-1/2}$

or $20x^3 - \frac{6}{x^3} + \frac{3}{\sqrt{x}}$

b) $y = \frac{x^3}{2x+1}$

$\frac{dy}{dx} = \frac{(2x+1)(3x^2) - x^3(2)}{(2x+1)^2}$

$= \frac{4x^3 + 3x^2}{(2x+1)^2}$

or $\frac{x^2(4x+3)}{(2x+1)^2}$

Ex: Find $f'(1)$ for $f(x) = x^2(x^2+5x+1)(x^7+x^3+6)$

$f(x) = (x^4 + 5x^3 + x^2)(x^7 + x^3 + 6)$

$f'(x) = (x^4 + 5x^3 + x^2)(7x^6 + 3x^2) + (x^7 + x^3 + 6)(4x^3 + 15x^2 + 2x)$

$f'(1) = 7(10) + 8(21)$

$= 238$

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\csc x$ | $-\csc x \cot x$ |
| $\cot x$ | $-\csc^2 x$ |

Ex: Find $y'$

a) $y = \csc x^2$

$y' = -\csc x^2 \cot x^2 \, (2x)$

$= -2x \csc x^2 \cot x^2$

b) $y = \csc^2 x$

$y = [\csc x]^2$

$y' = 2[\csc x](-\csc x \cot x)$

$= -2\csc^2 x \cot x$

Ex: Why does $\frac{d}{dx}[\tan x] = \sec^2 x$?

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

$$= \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \left(\frac{1}{\cos x}\right)^2$$

$$= \sec^2 x$$

Ex: Find $\frac{df}{dx}$

a) $f = \sin 5x$

$$\frac{df}{dx} = 5\cos 5x$$

b) $f = x\tan x^2$

$$\frac{df}{dx} = x\sec^2 x^2(2x) + \tan x^2(1)$$

$$= 2x^2\sec^2 x^2 + \tan x^2$$

c) $f = \sin^3 x + \cos^3 x$

$$\frac{df}{dx} = 3\sin^2 x\cos x + 3\cos^2 x(-\sin x)$$

$$= 3\sin x\cos x(\sin x - \cos x)$$

d) $f = \sec^2 x^3$

$f = [\sec x^3]^2$

$\frac{df}{dx} = 2\sec x^3 \, [\sec x^3 \tan x^3 (3x^2)]$

$= 6x^2 \sec^2 x^3 \tan x^3$

Ex: Find the second derivative of $y = \sin 3x^2$

$y' = \cos 3x^2 (6x)$

$= \underbrace{6x}\,\underbrace{\cos 3x^2}$

$y'' = 6x[-\sin 3x^2 (6x)] + \cos 3x^2 (6)$

$= -36x^2 \sin 3x^2 + 6\cos 3x^2$

Ex: Find an equation for the tangent line to $y = 2x^3 + 5x^2 - 1$ at $x = 1$

$y' = 6x^2 + 10x$

$y'|_{x=1} = 16$

$m = 16$ $\quad x = 1$

$\Rightarrow y = 6$

$y - y_1 = m(x - x_1)$

$y - 6 = 16(x - 1)$

$y - 6 = 16x - 16$

$y = 16x - 10$