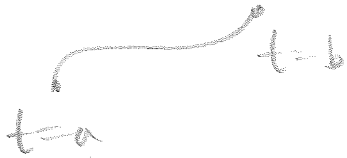
12.5 Arc length



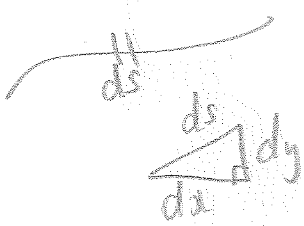
s = arc length
(length of curve)

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \qquad (2D)$$

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt \qquad (3D)$$

$$\text{or } s = \int_a^b \|\vec{v}(t)\|\, dt \qquad (2D \text{ and } 3D)$$

Why?



$$ds = \sqrt{dx^2 + dy^2}$$

$$= \sqrt{\left(\frac{dx^2}{dt^2} + \frac{dy^2}{dt^2}\right) dt^2}$$

$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Ex: Find the arc length of

$\vec{r}(t) = [\cos 3t, \sin 3t, 2t]$ on $0 \le t \le 4\pi$

$\vec{r}'(t) = [-3\sin 3t, 3\cos 3t, 2]$

$$S = \int_a^b \|\vec{r}'(t)\| \, dt$$

$$= \int_0^{4\pi} \sqrt{9\sin^2 3t + 9\cos^2 3t + 4} \; dt$$

$$= \int_0^{4\pi} \sqrt{9+4} \; dt$$

$$= \int_0^{4\pi} \sqrt{13} \, dt$$

$$= \sqrt{13}\, t \Big|_0^{4\pi}$$

$$= 4\sqrt{13}\,\pi$$