

12.1 Vector-Valued Functions

Recall: parametric curve (Section 12.2)

$$\begin{cases} x = x(t) \\ y = y(t) \\ a \leq t \leq b \end{cases}$$

Can be described with the position vector

$$\vec{r}(t) = [x(t), y(t)] \quad (a \leq t \leq b)$$

$$\text{or } \vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} \quad (a \leq t \leq b)$$

Ex: $\vec{r}(t) = [t, t^2]$

Find $\vec{r}(6)$

$$\vec{r}(6) = [6, 36]$$

Note: $\vec{r}(t)$ is called a vector-valued function.

Ex: Find the position vector:

$$\begin{cases} x = 4 + 2t \\ y = 1 + 5t \\ z = 3 + t \\ 0 \leq t \leq 1 \end{cases}$$

$$\vec{r}(t) = [4 + 2t, 1 + 5t, 3 + t] \quad (0 \leq t \leq 1)$$

$$\text{or } \vec{r}(t) = (4 + 2t)\vec{i} + (1 + 5t)\vec{j} + (3 + t)\vec{k} \quad (0 \leq t \leq 1)$$

Ex: Find the position vector of the
line segment from $(1, 2, 3)$ to $(-4, 6, 8)$

x : Starts at 1, net change of -5

y : 2, 4

z : 3, 5

$$\begin{cases} x = 1 - 5t \\ y = 2 + 4t \\ z = 3 + 5t \\ 0 \leq t \leq 1 \end{cases}$$

$$r(t) = [1 - 5t, 2 + 4t, 3 + 5t] \quad (0 \leq t \leq 1)$$