

## 10.2 Parametric Curves

Parametric curve

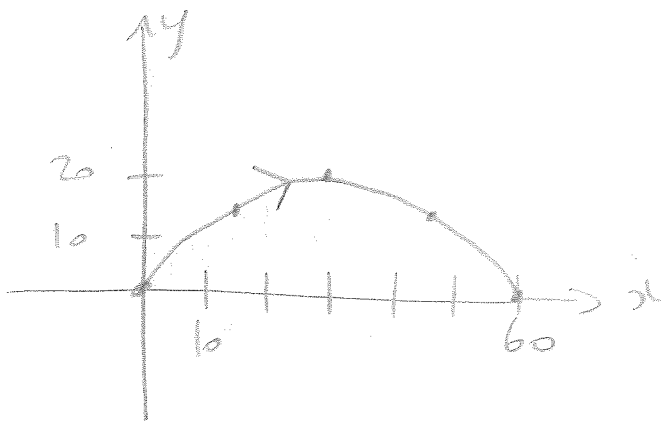
$$\begin{cases} x = f(t) \\ y = g(t) \\ a \leq t \leq b \end{cases}$$

Ex: Plot

$$\begin{cases} x = 15t \\ y = 20t - 5t^2 \\ 0 \leq t \leq 4 \end{cases}$$

t	x	y
0	0	0
1	15	15
2	30	20
3	45	15
4	60	0

Plot



Ex: Eliminate the parameter

$$\begin{cases} x = 15t \\ y = 20t - 5t^2 \end{cases}$$

$$\frac{x}{15} = t \Rightarrow y = 20\left(\frac{x}{15}\right) - 5\left(\frac{x}{15}\right)^2$$

$$y = \frac{4}{3}x - \frac{5x^2}{225}$$

$$\text{or } y = \frac{4}{3}x - \frac{x^2}{45}$$

Ex: Eliminate the parameter

$$\begin{cases} x = h + r \cos t \\ y = k + r \sin t \\ 0 \leq t < 2\pi \end{cases}$$

$$x - h = r \cos t$$

$$\frac{x - h}{r} = \cos t$$

$$\frac{y - k}{r} = \sin t$$

$$\cos^2 t + \sin^2 t = 1$$

$$\left(\frac{x - h}{r}\right)^2 + \left(\frac{y - k}{r}\right)^2 = 1$$

$$(x - h)^2 + (y - k)^2 = r^2$$

CIRCLE



Ex: Eliminate the parameter

$$\begin{cases} x = h + a \cos t \\ y = k + b \sin t \\ 0 \leq t < 2\pi \end{cases}$$

$$\frac{x-h}{a} = \cos t$$

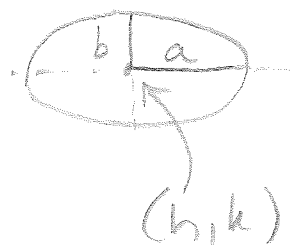
$$\frac{y-k}{b} = \sin t$$

$$\cos^2 t + \sin^2 t = 1$$

$$\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

ELLIPSE



Ex: Write as a parametric curve

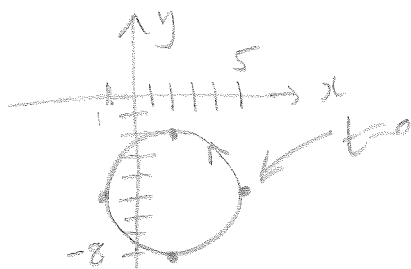
$$(x-2)^2 + (y+5)^2 = 9$$

$$\frac{(x-2)^2}{9} + \frac{(y+5)^2}{9} = 1$$

$$\frac{x-2}{3} = \cos t \Rightarrow x = 2 + 3 \cos t$$

$$\frac{y+5}{3} = \sin t \Rightarrow y = -5 + 3 \sin t$$

$0 \leq t < 2\pi$



Parametrization of a curve is not unique.

Ex: Give three parametrizations of the line segment from  $(2, 5)$  to  $(4, 8)$

a) 
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{cases} x = 2 + 2t \\ y = 5 + 3t \\ 0 \leq t \leq 1 \end{cases}$$

b) Speed it up

$$\begin{cases} x = 2 + 4t \\ y = 5 + 6t \\ 0 \leq t \leq 0,5 \end{cases}$$

c) Slow it down

$$\begin{cases} x = 2 + t \\ y = 5 + 1.5t \\ 0 \leq t \leq 2 \end{cases}$$