1. [5 marks] Find the sum:

$$\sum_{n=1}^{\infty}\left[\frac{7}{4^n} + \left(\frac{2}{5}\right)^{n+1}\right]$$

geometric
$a = \frac{7}{4}$
$r = \frac{1}{4}$

geometric
$a = \frac{4}{25}$
$r = \frac{2}{5}$

$$= \frac{\left(\frac{7}{4}\right)}{\left(\frac{3}{4}\right)} + \frac{\left(\frac{4}{25}\right)}{\left(\frac{3}{5}\right)}$$

$$= \frac{7}{3} + \frac{4}{25} \cdot \frac{5}{3}$$

$$= \frac{7}{3} + \frac{4}{15}$$

$$= \frac{39}{15} \text{ or } \frac{13}{5} \text{ or } 2.6$$

2. [3 marks] Use the Direct Comparison Test to decide whether the series below converges or diverges. Show all your work.

$$\sum_{n=1}^{\infty} \frac{0.4+0.5|\sin n|}{n^{1.5}}$$

$$0 < \frac{0.4+0.5|\sin n|}{n^{1.5}} \leq \frac{1}{n^{1.5}} \text{ for } n \geq 1$$

and $\sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$ converges

$$\Rightarrow \sum_{n=1}^{\infty} \frac{0.4+0.5|\sin n|}{n^{1.5}} \text{ converges}$$

3. [6 marks] a) Find the 2nd degree Taylor polynomial of $f(x) = \frac{4}{x}$ centred at $c = 2$.

$f(x) = 4x^{-1}$ $\qquad$ $f(2) = 2$

$f'(x) = -4x^{-2}$ $\qquad$ $f'(2) = -1$

$f''(x) = 8x^{-3}$ $\qquad$ $f''(2) = 1$

$$P_2(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2$$

$$= 2 \quad - (x-2) + \frac{1}{2}(x-2)^2$$

b) Find an upper bound for the error $|R_2(2.3)|$.

$f'''(x) = -24x^{-4}$

$f'''(z) = \frac{-24}{z^4}$

$$|R_2(x)| = \left| \frac{f'''(z)}{3!}(x-c)^3 \right|$$

$$= \frac{1}{6}\left(\frac{24}{z^4}\right)(0.3)^3$$

for z between 2 and 2.3

$$\leq \frac{1}{6}\left(\frac{24}{2^4}\right)(0.3)^3$$

$$\leq 0.00675$$

4. [5 marks] Find the interval of convergence:

$$\sum_{n=1}^{\infty} \frac{(x-2)^{n-1}}{n \cdot 8^{n-1}}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x-2)^n}{(n+1)8^n} \cdot \frac{n\,8^{n-1}}{(x-2)^{n-1}}\right|$$

$$= \left|\frac{(x-2)}{8} \cdot \frac{n}{n+1}\right|$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x-2}{8}\right|$$

Series converges if $\left|\frac{x-2}{8}\right| < 1$

$|x-2| < 8$

$-8 < x-2 < 8$

$-6 < x < 10$

$x = -6$: Series $= \sum_{n=1}^{\infty} \frac{(-8)^{n-1}}{n \cdot 8^{n-1}}$

$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}\cancel{8^{n-1}}}{n \cdot \cancel{8^{n-1}}}$

Converges (Alternating Series Test)

$x = 10$: Series $= \sum_{n=1}^{\infty} \frac{\cancel{8^{n-1}}}{n \cdot \cancel{8^{n-1}}}$

Diverges (p-series)

$\boxed{-6 \le x < 10}$

5. [3 marks] $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ converges by the Alternating Series Test.

a) Find S_4. Round your answer to four decimal places.

$$S_4 = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16}$$

$$\approx 0.7986$$

b) Find an upper bound for $|R_4|$.

$$|R_4| \leq a_5$$

$$\leq \frac{1}{25}$$

$$\leq 0.04$$

c) Estimate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ using parts a) and b).

$$0.7986 - 0.04 \leq \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \leq 0.7986 + 0.04$$

6. [3 marks] $\sum_{n=1}^{\infty} \frac{1}{n^{3.5}}$ converges by the Integral Test. Find the smallest N so that $R_N \leq 0.02$

$$\int_N^\infty \frac{1}{x^{3.5}}\,dx$$

$$= \lim_{b\to\infty} \int_N^b x^{-3.5}\,dx$$

$$= \lim_{b\to\infty} \frac{-1}{2.5} x^{-2.5} \Big|_N^b$$

$$= \lim_{b\to\infty} \frac{-1}{2.5} b^{-2.5} + \frac{1}{2.5} N^{-2.5}$$

$$= \frac{1}{2.5} N^{-2.5}$$

Let $\frac{1}{2.5} N^{-2.5} \leq 0.02$

$$\frac{1}{2.5(0.02)} \leq N^{2.5}$$

$$\left(\frac{1}{2.5(0.02)}\right)^{\frac{1}{2.5}} \leq N$$

$$N \geq 3.3$$

$$\boxed{N = 4}$$