1. [2 marks] Consider the sequence below:

$a_n = \frac{8n+\sqrt{n}}{9n}$ for $1 \leq n < \infty$

a) Find the first two terms.

$a_2 = \frac{16+\sqrt{2}}{18}$ $\quad$ $a_3 = \frac{24+\sqrt{3}}{27}$

b) Find the sequence's limit (if it exists).

$$\lim_{n\to\infty} \frac{8n+\sqrt{n}}{9n} \leftarrow \frac{\infty}{\infty}$$

$$= \lim_{n\to\infty} \frac{8 + \frac{1}{2}n^{-1/2}}{9}$$

$$= \frac{8+0}{9}$$

$$= \frac{8}{9}$$

2. [3 marks] Evaluate $\int x^2 \cos 3x \, dx$

Integration By Parts

	D	I
⊕	x^2	$\cos 3x$
⊖	$2x$	$\frac{\sin 3x}{3}$
⊕	2	$-\frac{\cos 3x}{9}$
		$-\frac{\sin 3x}{27}$

$$\text{Integral} = \frac{x^2 \sin 3x}{3} + \frac{2x \cos 3x}{9} - \frac{2 \sin 3x}{27} + C$$

3. [3 marks] Evaluate $\int (1 + 2\sin 3\theta)^2 \, d\theta$

$$= \int (1 + 4\sin 3\theta + 4\sin^2 3\theta)\, d\theta$$

$$= \int 1 + 4\sin 3\theta + 2 - 2\cos 6\theta)\, d\theta$$

$$= \theta - \frac{4\cos 3\theta}{3} + 2\theta - \frac{2\sin 6\theta}{6} + C$$

$$= 3\theta - \frac{4\cos 3\theta}{3} - \frac{\sin 6\theta}{3} + C$$

4. [3 marks] Evaluate or show that it diverges: $\int_4^5 \frac{1}{(x-4)^2}\,dx$

$$= \lim_{t \to 4^+} \int_t^5 \frac{1}{(x-4)^2}\,dx$$

$$= \lim_{t \to 4^+} -(x-4)^{-1} \Big|_t^5$$

$$= \lim_{t \to 4^+} -1 + \frac{1}{t-4}$$

$$= -1 + \infty$$

$$= \infty$$

The integral diverges.

5. [4 marks] Evaluate $\lim_{x\to\infty}(1+\frac{3}{x})^x$

Let $L = \lim_{x\to\infty}(1+\frac{3}{x})^x$

$\ln L = \lim_{x\to\infty} \ln(1+\frac{3}{x})^x$

$= \lim_{x\to\infty} x \ln(1+\frac{3}{x})$ (1)

$= \lim_{x\to\infty} \frac{\ln(1+\frac{3}{x})}{(\frac{1}{x})}$ ← $\frac{0}{0}$ (1)

$= \lim_{x\to\infty} \frac{\frac{1}{(1+\frac{3}{x})}(\frac{-3}{x^2})}{(\frac{-1}{x^2})}$ (1)

$= \lim_{x\to\infty} \frac{3}{1+\frac{3}{x}}$

$= 3$ (1)

$\ln L = 3 \Rightarrow L = e^3$

6. [5 marks] Evaluate $\int \frac{6}{x(x+7)^2}\,dx$

$$\frac{6}{x(x+7)^2} = \frac{A}{x} + \frac{B}{x+7} + \frac{C}{(x+7)^2} \quad (1)$$

$$\boxed{6 = A(x+7)^2 + Bx(x+7) + Cx}$$

Sub $x=0$: $6 = 49A \Rightarrow A = \frac{6}{49}$

$x=-7$: $6 = -7C \Rightarrow C = \frac{-6}{7}$

x^2 coefficient: $0 = A+B \Rightarrow B = \frac{-6}{49}$ } (2)

$$\text{Integral} = \int \left[\frac{6}{49}\frac{1}{x} - \frac{6}{49}\frac{1}{x+7} - \frac{6}{7}\frac{1}{(x+7)^2}\right]dx$$

$$= \frac{6}{49}\ln|x| - \frac{6}{49}\ln|x+7| + \frac{6}{7}(x+7)^{-1} + C \quad (2)$$

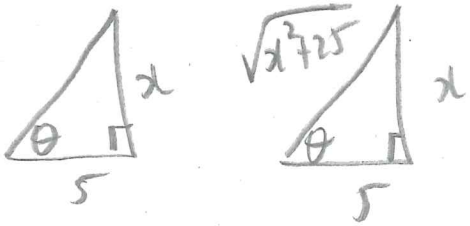
7. [5 marks] Evaluate $\int \frac{dx}{\sqrt{x^2+25}^3}$

(1)

Sub $x = 5\tan\theta$

$dx = 5\sec^2\theta\, d\theta$ (1)

$\frac{x}{5} = \tan\theta$



$\frac{\sqrt{x^2+25}}{5} = \sec\theta$

$\sqrt{x^2+25} = 5\sec\theta$ (1)

$\text{Integral} = \int \frac{5\sec^2\theta\, d\theta}{(5\sec\theta)^3}$

$= \frac{1}{25}\int \frac{1}{\sec\theta}\, d\theta$

$= \frac{1}{25}\int \cos\theta\, d\theta$ (1)

$= \frac{1}{25}\sin\theta + C$

$= \frac{1}{25}\cdot\frac{x}{\sqrt{x^2+25}} + C$ (1)