1. [4 marks] Given $\vec{r}(t) = [8\sqrt{t}, \frac{3}{t}, 2t - 4]$. Find:

a) $\vec{r}\,'(t) \cdot \vec{r}\,''(t)$

$$\vec{r}\,'(t) = [4t^{-1/2}, -3t^{-2}, 2]$$

$$\vec{r}\,''(t) = [-2t^{-3/2}, 6t^{-3}, 0]$$

$$\vec{r}\,'(t) \cdot \vec{r}\,''(t) = -8t^{-2} - 18t^{-5}$$

b) $\vec{r}\,'(t) \times \vec{r}\,''(t)$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4t^{-1/2} & -3t^{-2} & 2 \\ -2t^{-3/2} & 6t^{-3} & 0 \end{vmatrix}$$

$$= \vec{i}(-12t^{-3}) - \vec{j}(4t^{-3/2}) + \vec{k}(24t^{-7/2} - 6t^{-7/2})$$

$$= [-12t^{-3}, -4t^{-3/2}, 18t^{-7/2}]$$

2. [3 marks] Find $\frac{d^2y}{dx^2}$ given:

$$\begin{cases} x = 8t^3 + 2 \\ y = 2t^2 + 4t + 1 \end{cases}$$

$$\frac{dx}{dt} = 24t^2$$

$$\frac{dy}{dt} = 4t + 4$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

$$= \frac{4t+4}{24t^2}$$

$$= \frac{1}{6t} + \frac{1}{6t^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\left(\frac{dx}{dt}\right)}$$

$$= \frac{\left(-\frac{1}{6}t^{-2} - \frac{1}{3}t^{-3}\right)}{(24t^2)}$$

$$= -\frac{1}{144}t^{-4} - \frac{1}{72}t^{-5}$$

3. [3 marks] Rewrite as a power series centred at $c = -1$:

$f(x) = \frac{3}{7x+6}$

$$= \frac{3}{7(x+1) - 1}$$

$$= \frac{-3}{1 - 7(x+1)}$$

$$= -3 \left[\frac{1}{1 - 7(x+1)} \right]$$

$$= -3 \sum_{n=0}^{\infty} [7(x+1)]^n$$

4. [5 marks] Use three nonzero terms of an appropriate series to approximate $\int_0^{0.8} x^3\sqrt{1+x^3}\,dx$.

$$(1+x)^{1/2} \approx 1 + \frac{1}{2}x + \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)x^2$$

$$\approx 1 + \frac{x}{2} - \frac{x^2}{8}$$

$$(1+x^3)^{1/2} \approx 1 + \frac{x^3}{2} - \frac{x^6}{8}$$

$$x^3(1+x^3)^{1/2} \approx x^3 + \frac{x^6}{2} - \frac{x^9}{8}$$

$$\int_0^{0.8} x^3(1+x^3)^{1/2}\,dx \approx \int_0^{0.8} \left(x^3 + \frac{x^6}{2} - \frac{x^9}{8}\right)dx$$

$$\approx \left[\frac{x^4}{4} + \frac{x^7}{14} - \frac{x^{10}}{80}\right]_0^{0.8}$$

$$\approx 0.12$$

5. [4 marks] Find $\frac{dy}{dx}$ given $r = 1 - \cos\theta$.

$$x = r\cos\theta$$
$$= (1-\cos\theta)\cos\theta$$
$$= \cos\theta - \cos^2\theta$$

$$\frac{dx}{d\theta} = -\sin\theta - 2\cos\theta(-\sin\theta)$$
$$= -\sin\theta + 2\sin\theta\cos\theta$$

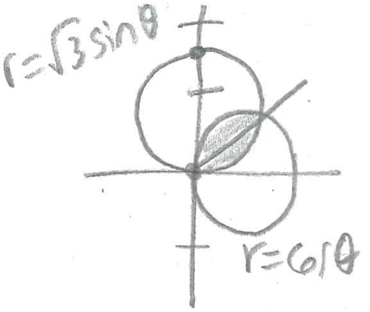
$$y = r\sin\theta$$
$$= (1-\cos\theta)\sin\theta$$

$$\frac{dy}{d\theta} = (1-\cos\theta)\cos\theta + \sin\theta(\sin\theta)$$
$$= \cos\theta - \cos^2\theta + \sin^2\theta$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$$

$$= \frac{\cos\theta - \cos^2\theta + \sin^2\theta}{2\sin\theta\cos\theta - \sin\theta}$$

6. [6 marks] Find the area inside both circles: $r = \sqrt{3}\sin\theta$ and $r = \cos\theta$.



Intersection : $r = r$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$ (1)

$A = A_1 + A_2$ where

$$A_1 = \frac{1}{2}\int_0^{\pi/6} (\sqrt{3}\sin\theta)^2\, d\theta$$ (1)

$$= \frac{3}{2}\int_0^{\pi/6} \sin^2\theta\, d\theta$$

$$= \frac{3}{2}\int_0^{\pi/6} \left(\frac{1}{2} - \frac{\cos 2\theta}{2}\right) d\theta$$

$$= \frac{3}{2}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/6}$$

$$= \frac{3}{2}\left[\frac{\pi}{12} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{\pi}{8} - \frac{3\sqrt{3}}{16}$$ (1)

$$A_2 = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$ (1)

$\rightarrow$

$$= \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{\cos 2\theta}{2} \right) d\theta$$

$$= \frac{1}{2} \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_{\pi/6}^{\pi/2}$$

$$= \frac{1}{2} \left[\frac{\pi}{4} - \left(\frac{\pi}{12} + \frac{1}{4} \frac{\sqrt{3}}{2} \right) \right]$$

$$= \frac{1}{2} \left[\frac{\pi}{6} - \frac{\sqrt{3}}{8} \right]$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{16}$$

1

$$A = A_1 + A_2$$

$$= \frac{\pi}{8} - \frac{3\sqrt{3}}{16} + \frac{\pi}{12} - \frac{\sqrt{3}}{16}$$

1

$$= \frac{5\pi}{24} - \frac{\sqrt{3}}{4}$$