1. [5 marks] Find the sum:

$$\sum_{n=1}^{\infty}\left[\frac{7}{3^n} + \left(\frac{3}{5}\right)^{n+1}\right]$$

geometric
$a = \frac{7}{3}$
$r = \frac{1}{3}$

geometric
$a = \frac{9}{25}$
$r = \frac{3}{5}$

$$= \frac{\left(\frac{7}{3}\right)}{\left(\frac{2}{3}\right)} + \frac{\left(\frac{9}{25}\right)}{\left(\frac{2}{5}\right)}$$

$$= \frac{7}{2} + \frac{9}{25}\cdot\frac{5}{2}$$

$$= \frac{7}{2} + \frac{9}{10}$$

$$= \frac{44}{10} \text{ or } \frac{22}{5} \text{ or } 4.4$$

2. [3 marks] Use the Direct Comparison Test to decide whether the series below converges or diverges. Show all your work.

$$\sum_{n=1}^{\infty} \frac{0.3+0.6|\sin n|}{n^{1.2}}$$

$$0 < \frac{0.3+0.6|\sin n|}{n^{1.2}} \le \frac{1}{n^{1.2}} \quad \text{for } n \geq 1$$

and $\sum_{n=1}^{\infty} \frac{1}{n^{1.2}}$ converges

$$\Rightarrow \sum_{n=1}^{\infty} \frac{0.3+0.6|\sin n|}{n^{1.2}} \text{ converges}$$

3. [5 marks] a) Find the 2nd degree Taylor polynomial of $f(x) = \frac{8}{x}$ centred at $c = 2$.

$f(x) = 8x^{-1}$ $\quad$ $f(2) = 4$

$f'(x) = -8x^{-2}$ $\quad$ $f'(2) = -2$

$f''(x) = 16x^{-3}$ $\quad$ $f''(2) = 2$

$$P_2(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2$$

$$= 4 - 2(x-2) + (x-2)^2$$

b) Find an upper bound for the error $|R_2(2.1)|$.

$f'''(x) = -48x^{-4}$

$f'''(z) = \frac{-48}{z^4}$

$$|R_2(x)| = \left| \frac{f'''(z)}{3!}(x-c)^3 \right|$$

$$|R_2(2.1)| = \frac{1}{6}\frac{48}{z^4}(0.1)^3$$

where z is between 2 and 2.1

$$\leq \frac{1}{6}\left(\frac{48}{2^4}\right)(0.1)^3$$

$$\leq 0.0005$$

4. [5 marks] Find the interval of convergence:
$\sum_{n=1}^{\infty} \frac{(x-3)^{n-1}}{n \cdot 9^{n-1}}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x-3)^n}{(n+1)\cdot 9^n} \cdot \frac{n \cdot 9^{n-1}}{(x-3)^{n-1}}\right|$$

$$= \left|\frac{(x-3)}{9} \cdot \frac{n}{n+1}\right|$$

$$\boxed{\begin{array}{l} a_n = \dfrac{(x-3)^{n-1}}{n \cdot 9^{n-1}} \\ a_{n+1} = \dfrac{(x-3)^n}{(n+1)9^n} \end{array}}$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x-3}{9}\right|$$

Series Converges if $\left|\frac{x-3}{9}\right| < 1$

$$|x-3| < 9$$

$$-9 < x-3 < 9$$

$$-6 < x < 12$$

$x=-6$: Series $= \sum_{n=1}^{\infty} \frac{(-9)^{n-1}}{n \cdot 9^{n-1}} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cancel{9^{n-1}}}{n \cdot \cancel{9^{n-1}}}$

converges (Alternating Series Test)

$x=12$: Series $= \sum_{n=1}^{\infty} \frac{\cancel{9^{n-1}}}{n \cdot \cancel{9^{n-1}}}$ diverges (p-series)

$$\boxed{-6 \le x < 12}$$

5. [3 marks] $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ converges by the Alternating Series Test.

a) Find S_4. Round your answer to four decimal places.

$$S_4 = 1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64}$$

$$\approx 0.8964$$

b) Find an upper bound for $|R_4|$

$$|R_4| \leq a_5$$

$$\leq \frac{1}{125}$$

$$\leq 0.008$$

c) Estimate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ using parts a) and b).

$$0.8964 - 0.008 \leq \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n3} \leq 0.8964 + 0.008$$

6. [3 marks] $\sum_{n=1}^{\infty} \frac{1}{n^{2.5}}$ converges by the Integral Test. Find the smallest N so that $R_N \leq 0.1$

$$\int_N^{\infty} \frac{1}{x^{2.5}}\,dx$$

$$= \lim_{b\to\infty} \int_N^b x^{-2.5}\,dx$$

$$= \lim_{b\to\infty} \frac{-1}{1.5} x^{-1.5} \Big|_N^b$$

$$= \lim_{b\to\infty} \frac{-1}{1.5} b^{-1.5} + \frac{1}{1.5} N^{-1.5}$$

$$= \frac{1}{1.5} N^{-1.5}$$

Let $\frac{1}{1.5} N^{-1.5} \leq 0.1$

$$\frac{1}{0.1(1.5)} \leq N^{1.5}$$

$$\left(\frac{1}{0.1(1.5)}\right)^{\frac{1}{1.5}} \leq N$$

$$N \geq 3.5$$

$$\boxed{N=4}$$