1. [2 marks] Consider the sequence below:

$a_n = \frac{7n+\sqrt{n}}{8n}$ for $1 \leq n < \infty$

a) Find the first two terms.

$a_2 = \frac{14+\sqrt{2}}{16}$ $\quad$ $a_3 = \frac{21+\sqrt{3}}{24}$

b) Find the sequence's limit (if it exists).

$$\lim_{n\to\infty} \frac{7n+\sqrt{n}}{8n} \longleftarrow \frac{\infty}{\infty}$$

$$= \lim_{n\to\infty} \frac{7 + \frac{1}{2}n^{-1/2}}{8}$$

$$= \frac{7+0}{8}$$

$$= \frac{7}{8}$$

2. [3 marks] Evaluate $\int x^2 \sin 5x \, dx$

Integration By Parts

	D	I
+	x^2	$\sin 5x$
−	$2x$	$-\frac{\cos 5x}{5}$
+	2	$-\frac{\sin 5x}{25}$
		$\frac{\cos 5x}{125}$

$$\int x^2 \sin 5x \, dx = -\frac{x^2 \cos 5x}{5} + \frac{2x \sin 5x}{25} + \frac{2 \cos 5x}{125} + C$$

3. [3 marks] Evaluate $\int (1 + 2\cos 3\theta)^2 \, d\theta$

$$= \int (1 + 4\cos 3\theta + 4\cos^2 3\theta)\, d\theta$$

$$= \int \left(1 + 4\cos 3\theta + 2 + 2\cos 6\theta\right) d\theta$$

$$= \theta + \frac{4\sin 3\theta}{3} + 2\theta + \frac{2\sin 6\theta}{6} + C$$

$$= 3\theta + \frac{4\sin 3\theta}{3} + \frac{\sin 6\theta}{3} + C$$

4. [3 marks] Evaluate or show that it diverges: $\int_2^3 \frac{1}{(x-2)^2}\,dx$

$$= \lim_{t \to 2^+} \int_t^3 \frac{1}{(x-2)^2}\,dx$$

$$= \lim_{t \to 2^+} -(x-2)^{-1} \Big|_t^3$$

$$= \lim_{t \to 2^+} -1 + \frac{1}{t-2}$$

$$= -1 + \infty$$

$$= \infty$$

The integral diverges.

5. [4 marks] Evaluate $\lim_{x\to\infty}(1+\frac{4}{x})^x$

Let $L = \lim_{x\to\infty}(1+\frac{4}{x})^x$

$\ln L = \lim_{x\to\infty} \ln(1+\frac{4}{x})^x$

$= \lim_{x\to\infty} x\ln(1+\frac{4}{x})$ (1)

$= \lim_{x\to\infty} \frac{\ln(1+\frac{4}{x})}{(\frac{1}{x})}$ $\Leftarrow \frac{0}{0}$ (1)

$= \lim_{x\to\infty} \frac{\frac{1}{(1+\frac{4}{x})}\left(\frac{-4}{x^2}\right)}{\left(\frac{-1}{x^2}\right)}$ (1)

$= \lim_{x\to\infty} \frac{4}{(1+\frac{4}{x})}$

$= 4$ (1)

$\ln L = 4 \Rightarrow L = e^4$

6. [5 marks] Evaluate $\int \frac{2}{x(x+3)^2}\,dx$

$$\frac{2}{x(x+3)^2} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{(x+3)^2} \quad (1)$$

$$\boxed{2 = A(x+3)^2 + Bx(x+3) + Cx}$$

Sub $x=0$: $2 = 9A \Rightarrow A = \frac{2}{9}$

$x=-3$: $2 = -3C \Rightarrow C = -\frac{2}{3}$

x^2 Coefficient: $0 = A+B \Rightarrow B = -\frac{2}{9}$ } (2)

$$\text{Integral} = \int \left[\frac{2}{9}\frac{1}{x} - \frac{2}{9}\frac{1}{x+3} - \frac{2}{3}\frac{1}{(x+3)^2}\right] dx$$

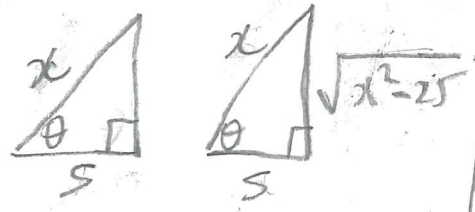
$$= \frac{2}{9}\ln|x| - \frac{2}{9}\ln|x+3| + \frac{2}{3}(x+3)^{-1} + C \quad (2)$$

7. [5 marks] Evaluate $\int \frac{dx}{x^2\sqrt{x^2-25}}$

Sub $x = 5\sec\theta$ (1)

$dx = 5\sec\theta\tan\theta\, d\theta$ (1)

$\frac{x}{5} = \sec\theta$



$\frac{\sqrt{x^2-25}}{5} = \tan\theta$

$\sqrt{x^2-25} = 5\tan\theta$ (1)

$\text{Integral} = \int \frac{5\sec\theta\tan\theta\, d\theta}{(5\sec\theta)^2(5\tan\theta)}$

$= \frac{1}{25}\int \frac{d\theta}{\sec\theta}$

$= \frac{1}{25}\int \cos\theta\, d\theta$ (1)

$= \frac{1}{25}\sin\theta + C$ } (1)

$= \frac{1}{25}\frac{\sqrt{x^2-25}}{x} + C$