26 $\frac{x}{2} = t$

$\frac{x}{2} = t \rightarrow \quad y = 4t - 12t^2$

$y = 4\left(\frac{x}{2}\right) - 12\left(\frac{x}{2}\right)^2$

$y = 2x - 12\left(\frac{x^2}{4}\right)$

$y = 2x - 3x^2$

(27) $\frac{dx}{dt} = 8\cos t(-\sin t)$
$= -8\cos t \sin t$

$\frac{dy}{dt} = 2\cos t$

Set $\frac{dx}{dt} = 0$: $-8\cos t \sin t = 0$

$-4\sin 2t = 0$

$\sin 2t = 0$

$2t = \ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots$

$t = \ldots, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \ldots$

But $0 \le t \le \frac{\pi}{2} \Rightarrow t = 0, \frac{\pi}{2}$

Set $\frac{dy}{dt} = 0$: $2\cos t = 0$

$\cos t = 0$

$t = \ldots, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

But $0 \le t \le \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2}$

Horizontal tangent when $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$.
No horizontal tangent.

Vertical tangent when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$

$t = 0$

$(x, y) = (4, 0)$

(28) $\frac{dx}{dt} = -\sin t \qquad \frac{dy}{dt} = 1-\cos t$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\sin^2 t + 1 - 2\cos t + \cos^2 t}$$
$$= \sqrt{2-2\cos t}$$
$$= \sqrt{2(1-\cos t)}$$

$$\boxed{\begin{aligned} \frac{1-\cos 2\theta}{2} &= \sin^2\theta \\ 1-\cos 2\theta &= 2\sin^2\theta \\ 1-\cos t &= 2\sin^2\frac{t}{2} \end{aligned}}$$

$$= \sqrt{4\sin^2\frac{t}{2}}$$
$$= \left|2\sin\frac{t}{2}\right|$$

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$
$$= \int_0^\pi \left|2\sin\frac{t}{2}\right| dt$$

$$\boxed{\sin\frac{t}{2} \geq 0 \quad \text{when } 0 \leq t \leq \pi}$$

$$= \int_0^\pi 2\sin\frac{t}{2}\, dt$$
$$= -4\cos\frac{t}{2}\Big|_0^\pi$$
$$= 4$$

(29) $r = 1 + \sin\theta$

$x = r\cos\theta$
$= \cos\theta + \cos\theta\sin\theta$
$= \cos\theta + \frac{1}{2}\sin 2\theta$

$\frac{dx}{d\theta} = -\sin\theta + \cos 2\theta$

Set $\frac{dx}{d\theta} = 0$:

$-\sin\theta + \cos 2\theta = 0$

$-\sin\theta + 1 - 2\sin^2\theta = 0$

$-2\sin^2\theta - \sin\theta + 1 = 0$

$(-2\sin\theta + 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$ $\qquad$ $\sin\theta = -1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

$y = r\sin\theta$
$= \sin\theta + \sin^2\theta$

$\frac{dy}{d\theta} = \cos\theta + 2\sin\theta\cos\theta$

Set $\frac{dy}{d\theta} = 0$:

$\cos\theta + 2\sin\theta\cos\theta = 0$

$\cos\theta(1 + 2\sin\theta) = 0$

$\cos\theta = 0$ $\qquad$ $\sin\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

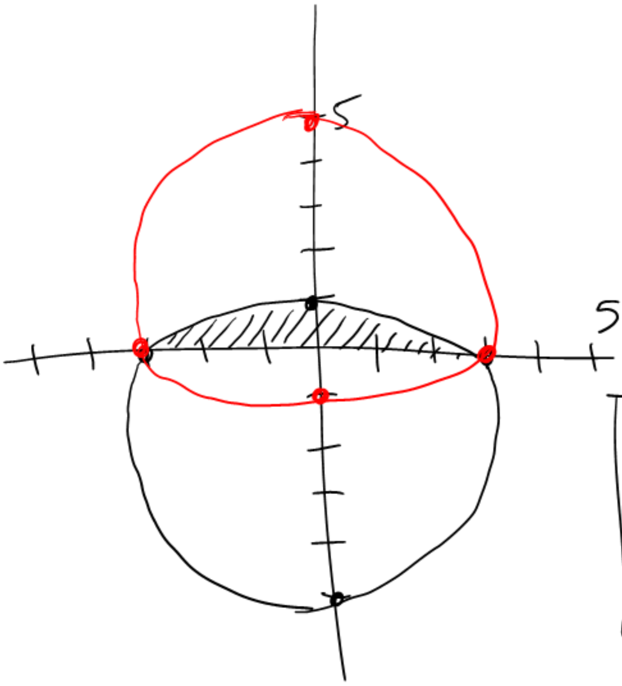
Vertical Tangent when $\frac{dx}{d\theta} = 0$ and $\frac{dy}{d\theta} \neq 0$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Horizontal Tangent when $\frac{dy}{d\theta} = 0$ and $\frac{dx}{d\theta} \neq 0$

$\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(30)



$r = 3 - 2\sin\theta$

$r = -3 + 2\sin\theta$

By symmetry, we can double the shaded area.

Use $r = 3 - 2\sin\theta$

for $0 \leq \theta \leq \pi$

$$A = 2 \cdot \frac{1}{2} \int_0^{\pi} (3 - 2\sin\theta)^2 \, d\theta$$

$$= \int_0^{\pi} (9 - 12\sin\theta + 4\sin^2\theta) \, d\theta$$

$$= \int_0^{\pi} (9 - 12\sin\theta + 2 - 2\cos 2\theta) \, d\theta$$

$$= \Big[11\theta + 12\cos\theta - \sin 2\theta \Big]_0^{\pi}$$

$$= (11\pi - 12) - (12)$$

$$= 11\pi - 24$$

(31) a) $\frac{d}{dt}\, 9t\,[t^2, t^3] = \frac{d}{dt}\,[9t^3, 9t^4]$

$= [27t^2, 36t^3]$

$\frac{d}{dt}\, 9t\,[t^2, t^3] = 9t\,[2t, 3t^2] + 9\,[t^2, t^3]$

$= [18t^2, 27t^3] + [9t^2, 9t^3]$

$= [27t^2, 36t^3]$

b) $\vec{r}(2t) = [14t+1, 8t]$

$\frac{d}{dt}\,\vec{r}(2t) = [14, 8]$

$\vec{r}\,'(t) = [7, 4]$

$\vec{r}\,'(2t) = [7, 4]$

$\frac{d}{dt}\,\vec{r}(2t) = \vec{r}\,'(2t)\,(2)$

$= [7, 4]\,(2)$

$= [14, 8]$

c) $\vec{r}\,'(t) = [3t^2, 7, 2t]$

$\vec{r}\,''(t) = [6t, 0, 2]$

$$\vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3t^2 & 7 & 2t \\ 6t & 0 & 2 \end{vmatrix}$$

$\rightarrow$

$= \vec{i}(14) - \vec{j}(-6t^2) + \vec{k}(-42t)$

$= [14, 6t^2, -42t]$

d) $\int_1^3 [6t^2, 8t]\,dt = [2t^3, 4t^2]\Big|_1^3$

$= [54, 36] - [2, 4]$

$= [52, 32]$

32 $\boxed{\vec{a}(t) = [4, 0, -9.8]}$

$\vec{v}(t) = [4t, 0, -9.8t] + \vec{C_1}$

Sub $t=0$: $[0, 25, 25] = [0, 0, 0] + \vec{C_1}$

$\vec{C_1} = [0, 25, 25]$

$\boxed{\vec{v}(t) = [4t, 25, -9.8t + 25]}$

$\vec{r}(t) = [2t^2, 25t, -4.9t^2 + 25t] + \vec{C_2}$

Sub $t=0$: $[0, 0, 0] = [0, 0, 0] + \vec{C_2}$

$\vec{C_2} = [0, 0, 0]$

$\boxed{\vec{r}(t) = [2t^2, 25t, -4.9t^2 + 25t]}$

It hits the ground when

(z-component of $\vec{r}$) $= 0$

$-4.9t^2 + 25t = 0$

$t(-4.9t + 25) = 0$

$t = 0, \frac{25}{4.9}$ $\boxed{t = \frac{25}{4.9}}$

$$\bar{v}\left(\frac{25}{4.9}\right) = \left[\frac{100}{4.9}, 25, -25\right]$$

Speed when it hits the ground

is $\left\|\bar{v}\left(\frac{25}{4.9}\right)\right\| \approx 40.8$ m/s

(33) $\vec{r} = [t, 1+t^{-1}, 0]$

$$\bar{v} = [1, -t^{-2}, 0]$$

$$\vec{a} = [0, 2t^{-3}, 0]$$

$$\bar{v} \cdot \bar{a} = -2t^{-5}$$

$$\|\bar{v}\| = \sqrt{1 + t^{-4}}$$

$$a_T = \frac{\bar{v} \cdot \bar{a}}{\|\bar{v}\|}$$

$$= \frac{-2t^{-5}}{\sqrt{1+t^{-4}}}$$

$$\bar{v} \times \bar{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -t^{-2} & 0 \\ 0 & 2t^{-3} & 0 \end{vmatrix}$$

$$= \vec{i}(0) - \vec{j}(0) + \vec{k}(2t^{-3})$$

$$= [0, 0, 2t^{-3}]$$

$$\|\bar{v} \times \bar{a}\| = \sqrt{4t^{-6}}$$
$$= |2t^{-3}|$$
$$= 2t^{-3} \quad \text{(assuming } t > 0\text{)}$$

$$a_N = \frac{\|\bar{v} \times \bar{a}\|}{\|\bar{v}\|}$$

$$= \frac{2t^{-3}}{\sqrt{1 + t^{-4}}}$$

(34)

$$\bar{r} = [\cos^3 t, \sin^3 t]$$
$$\bar{v} = [-3\cos^2 t \sin t, 3\sin^2 t \cos t]$$
$$\|\bar{v}\| = \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}$$
$$= \sqrt{9\cos^2 t \sin^2 t (\cos^2 t + \sin^2 t)}$$
$$= \sqrt{9\cos^2 t \sin^2 t}$$
$$= |3\cos t \sin t|$$
$$= 3\cos t \sin t \quad \text{on} \quad 0 \le t \le \frac{\pi}{2}$$

$$s = \int_0^{\frac{\pi}{2}} 3\cos t \sin t \, dt \quad \rightarrow$$

$$= \int_0^{\frac{\pi}{2}} \frac{3}{2} \sin 2t \, dt$$

$$= -\frac{3}{4} \cos 2t \Big|_0^{\pi/2}$$

$$= \frac{3}{4} - \left(-\frac{3}{4}\right)$$

$$= \frac{3}{2}$$