(11) $\int x^2 \ln x \, dx$

$$\boxed{\begin{array}{ll} u = \ln x & dv = x^2 dx \\ du = \frac{1}{x} dx & v = \frac{x^3}{3} \end{array}}$$

$$\begin{aligned} \int u\,dv &= uv - \int v\,du \\ &= \frac{x^3}{3}\ln x - \int \frac{x^2}{3} dx \\ &= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C \end{aligned}$$

(12) $\int \sec^4\theta \tan^3\theta \, d\theta$

Here are two methods. Both are correct.

Method 1

$$\int \sec^4\theta \tan^3\theta \, d\theta$$

$$= \int \sec^2\theta \sec^2\theta \tan^3\theta \, d\theta$$

$$= \int \sec^2\theta (1+\tan^2\theta) \tan^3\theta \, d\theta$$

$$= \int \sec^2\theta (\tan^3\theta + \tan^5\theta) \, d\theta$$

$$\boxed{\begin{array}{l} \text{Let } u = \tan\theta \\ du = \sec^2\theta \, d\theta \end{array}}$$

$$= \int (u^3 + u^5) \, du$$

$$= \frac{u^4}{4} + \frac{u^6}{6} + C$$

$$= \frac{\tan^4\theta}{4} + \frac{\tan^6\theta}{6} + C$$

Method 2

$$\int \sec^4\theta \tan^3\theta \, d\theta$$

$$= \int \sec^4\theta \tan^2\theta \tan\theta \, d\theta$$

$$= \int \sec^4\theta (\sec^2\theta - 1) \tan\theta \, d\theta$$

$$= \int \sec^3\theta (\sec^2\theta - 1) \sec\theta \tan\theta \, d\theta$$

$$\boxed{\begin{array}{l} \text{Let } u = \sec\theta \\ du = \sec\theta \tan\theta \, d\theta \end{array}}$$

$$= \int (u^5 - u^3) \, du$$

$$= \frac{u^6}{6} - \frac{u^4}{4} + C$$

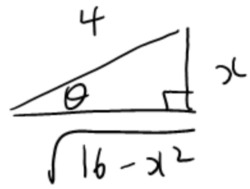
$$= \frac{\sec^6\theta}{6} - \frac{\sec^4\theta}{4} + C$$

(13) $$\int \frac{dx}{x^2\sqrt{16-x^2}}$$

Sub $x = 4\sin\theta$

$dx = 4\cos\theta \, d\theta$

$$\frac{x}{4} = \sin\theta$$



$$\frac{\sqrt{16-x^2}}{4} = \cos\theta$$

$$\sqrt{16-x^2} = 4\cos\theta$$

$$= \int \frac{4\cos\theta \, d\theta}{16\sin^2\theta\,(4\cos\theta)}$$

$$= \frac{1}{16}\int \csc^2\theta \, d\theta$$

$$= -\frac{1}{16}\cot\theta + C$$

$$= -\frac{1}{16}\,\frac{\sqrt{16-x^2}}{x} + C$$

(14) Let $\dfrac{13}{(x+2)(x^2+9)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+9}$

$$\boxed{13 = A(x^2+9) + (Bx+C)(x+2)}$$

Sub $x=-2$: $13 = 13A \Rightarrow A=1$

x^2 coefficient: $0 = A+B \Rightarrow B=-1$

Sub any x-value, say $x=0$:

$$13 = 9A + 2C$$
$$13 = 9 + 2C$$
$$C = 2$$

Integral $= \displaystyle\int \left[\frac{1}{x+2} - \frac{x}{x^2+9} + \frac{2}{x^2+9}\right] dx$

$= \ln|x+2| - \frac{1}{2}\ln|x^2+9| + \frac{2}{3}\tan^{-1}\frac{x}{3} + C$

(15) a) $\lim\limits_{x\to 0} \dfrac{\sin 6x}{\tan 7x}$ $\boxed{\text{The form is } \frac{0}{0}}$ ✓

$\overset{H}{=} \lim\limits_{x\to 0} \dfrac{6\cos 6x}{7\sec^2 7x}$

$= \dfrac{6}{7}$

b) Let $L = \lim\limits_{x\to 0^+} (e^x + 5x)^{1/x}$

$\ln L = \lim\limits_{x\to 0^+} \ln(e^x + 5x)^{1/x}$

$= \lim\limits_{x\to 0^+} \dfrac{1}{x} \ln(e^x + 5x)$

$= \lim\limits_{x\to 0^+} \dfrac{\ln(e^x + 5x)}{x}$

$\boxed{\text{The form is } \frac{0}{0}}$ ✓

$\overset{H}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{e^x+5x}(e^x+5)}{1}$

$= 6$

$L = e^{\ln L} = e^6$

(16) $$\int_0^\infty \frac{e^x}{1+(e^x)^2}\,dx$$

Let $u = e^x$

$du = e^x\,dx$

$x = 0 \Rightarrow u = 1$

$x \to \infty \Rightarrow u \to \infty$

$$= \int_1^\infty \frac{du}{1+u^2}$$

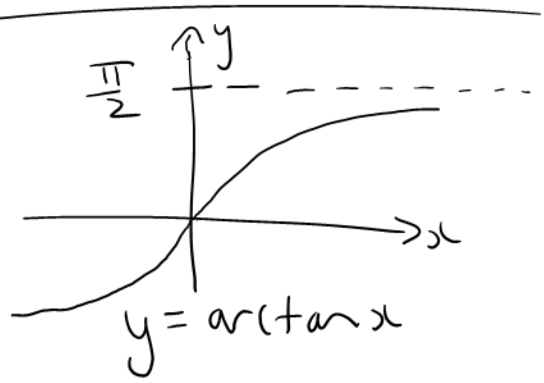
$$= \lim_{b\to\infty} \int_1^b \frac{du}{1+u^2}$$

$$= \lim_{b\to\infty} \arctan u \Big|_1^b$$

$$= \lim_{b\to\infty} \arctan b - \arctan 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4}$$



(17) a) $a_1 = e \qquad a_2 = \frac{e^2}{2} \qquad a_3 = \frac{e^3}{3}$

$$\lim_{n\to\infty} \frac{e^n}{n}$$

$\boxed{\text{Form is } \frac{\infty}{\infty} \checkmark}$

$$\overset{(H)}{=} \lim_{n\to\infty} \frac{e^n}{1}$$

$$= \infty$$

b) $a_0 = 0 \qquad a_1 = \frac{4}{\sqrt{2}} \qquad a_2 = \frac{8}{\sqrt{5}}$

$$\lim_{n\to\infty} \frac{4n}{\sqrt{n^2+1}}$$

$$= \lim_{n\to\infty} 4\sqrt{\frac{n^2}{n^2+1}}$$

$$= \lim_{n\to\infty} 4\sqrt{\frac{1}{1+\frac{1}{n^2}}}$$

$$= 4$$

(18) a) Partial Fractions

$$\frac{7}{(n+3)(n+4)} = \frac{A}{n+3} + \frac{B}{n+4}$$

$$7 = A(n+4) + B(n+3)$$

Sub $n=-3$: $7 = A$

$n=-4$: $7 = -B \Rightarrow B = -7$

$$\sum_{n=2}^{\infty} \frac{7}{(n+3)(n+4)} = \sum_{n=2}^{\infty} \left[\frac{7}{n+3} - \frac{7}{n+4}\right]$$ Telescoping

$$= \left(\frac{7}{5} - \frac{7}{6}\right) + \left(\frac{7}{6} - \frac{7}{7}\right) + \dots$$

$$= \frac{7}{5} - \lim_{n\to\infty} \frac{7}{n+4}$$

$$= \frac{7}{5}$$

b) $$\sum_{n=2}^{\infty} \frac{2^{n+1}}{7^n} = \frac{2^3}{7^2} + \frac{2^4}{7^3} + \dots$$

$$\boxed{\text{Geometric} \quad a = \frac{2^3}{7^2} = \frac{8}{49} \quad r = \frac{2}{7}}$$

$$= \frac{a}{1-r}$$

$$= \frac{\left(\frac{8}{49}\right)}{\left(\frac{5}{7}\right)}$$

$$= \frac{8(7)}{49(5)}$$

$$= \frac{8}{35}$$

(19) Plan: Calculate $\int_N^\infty \frac{2}{x^2}\,dx$

Solve $\int_N^\infty \frac{2}{x^2}\,dx \le 0{,}1$

$$\int_N^\infty \frac{2}{x^2}\,dx = \lim_{b\to\infty} \int_N^b \frac{2}{x^2}\,dx$$

$$= \lim_{b\to\infty} -\frac{2}{x}\Big|_N^b$$

$$= \lim_{b\to\infty} -\frac{2}{b} + \frac{2}{N}$$

$$= \frac{2}{N}$$

Solve $\frac{2}{N} \le 0.1$

$$2 \le 0.1N$$

$$20 \le N$$

20

a) $S_3 = -1 + \frac{1}{4} - \frac{1}{9}$

$= \frac{-31}{36}$

b) $|R_3| \leq a_4$

$\leq \frac{1}{4^2}$

$\leq \frac{1}{16}$

c) $\frac{-31}{36} - \frac{1}{16} \leq \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \leq \frac{-31}{36} + \frac{1}{16}$

$\frac{-133}{144} \leq \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \leq \frac{-115}{144}$