① $\lim_{x \to 3} \dfrac{\sqrt{x+1} - 2}{x-3}$

$= \lim_{x \to 3} \dfrac{(\sqrt{x+1} - 2)}{(x-3)} \cdot \dfrac{(\sqrt{x+1} + 2)}{(\sqrt{x+1} + 2)}$

$= \lim_{x \to 3} \dfrac{x+1 - 4}{(x-3)(\sqrt{x+1} + 2)}$

$= \lim_{x \to 3} \dfrac{(x-3)}{(x-3)(\sqrt{x+1} + 2)}$

$= \lim_{x \to 3} \dfrac{1}{\sqrt{x+1} + 2}$

$= \dfrac{1}{2+2}$

$= \dfrac{1}{4}$

(2) a) $y = 3x^4 \sec x$

$y' = 3x^4 \sec x \tan x + 12x^3 \sec x$

b) $y = \dfrac{3x^4}{\sec x}$

$y' = \dfrac{(\sec x)(12x^3) - 3x^4 \sec x \tan x}{\sec^2 x}$

or $y' = \dfrac{12x^3 - 3x^4 \tan x}{\sec x}$

c) $y = 3[\sec x]^4$

$y' = 12[\sec x]^3 \sec x \tan x$

$= 12 \sec^4 x \tan x$

d) $y = \sec(3x^4)$

$y' = \sec(3x^4) \tan(3x^4) (12x^3)$

$= 12x^3 \sec(3x^4) \tan(3x^4)$

(3) a) $y = \ln\left(x\sqrt{x^2-9}\right)$

$= \ln x + \ln\sqrt{x^2-9}$

$= \ln x + \frac{1}{2}\ln(x^2-9)$

$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\ \frac{2x}{x^2-9}$

$= \frac{1}{x} + \frac{x}{x^2-9}$

b) $y = \frac{e^{(x^3)}}{e^{7x}}$

$= e^{x^3-7x}$

$\frac{dy}{dx} = (3x^2-7)\,e^{x^3-7x}$

(4) $y = 3\arcsin e^{6x} + \sqrt{1-e^{12x}}$

$$y' = 3\,\frac{1}{\sqrt{1-(e^{6x})^2}}\,(6e^{6x}) + \frac{1}{2}(1-e^{12x})^{-1/2}(-12e^{12x})$$

$$= \frac{18e^{6x}}{\sqrt{1-e^{12x}}} - \frac{6e^{12x}}{\sqrt{1-e^{12x}}}$$

$$= \frac{18e^{6x}-6e^{12x}}{\sqrt{1-e^{12x}}}$$

5 $$(x^3+y^3)^2 = 2xy$$

$$2(x^3+y^3)[3x^2+3y^2y'] = 2xy' + 2y$$

$$6x^2(x^3+y^3) + 6y^2(x^3+y^3)y' = 2xy' + 2y$$

$$6y^2(x^3+y^3)y' - 2xy' = 2y - 6x^2(x^3+y^3)$$

$$[6y^2(x^3+y^3) - 2x]y' = 2y - 6x^2(x^3+y^3)$$

$$y' = \frac{2y - 6x^2(x^3+y^3)}{6y^2(x^3+y^3) - 2x}$$

or $$y' = \frac{y - 3x^5 - 3x^2y^3}{3x^3y^2 + 3y^5 - x}$$

6 a) $\displaystyle\int_0^1 \frac{7x}{\sqrt{5x^2+4}}\,dx$

$$\boxed{\begin{aligned} &\text{Let } u = 5x^2+4 \\ &du = 10x\,dx \\ &\frac{du}{10} = x\,dx \\ &x=0 \Rightarrow u=4 \\ &x=1 \Rightarrow u=9 \end{aligned}}$$

$$= \frac{7}{10}\int_4^9 \frac{du}{\sqrt{u}}$$

$$= \frac{7}{10}\int_4^9 u^{-1/2}\,du$$

$$= \frac{7}{10}\left[2u^{1/2}\right]_4^9$$

$$= \frac{7}{5}(3-2)$$

$$= \frac{7}{5}$$

b) $\displaystyle\int \frac{7x}{\sqrt{5x^2+4}}\,dx$

$$\boxed{\begin{aligned} &\text{Let } u = 5x^2+4 \\ &du = 10x\,dx \\ &\frac{du}{10} = x\,dx \end{aligned}}$$

$$= \frac{7}{10}\int \frac{du}{\sqrt{u}}$$

$$= \frac{7}{10} \int u^{-1/2}\, du$$

$$= \frac{7}{10} \left[2u^{1/2} \right] + C$$

$$= \frac{7}{5} \sqrt{5x^2+4} + C$$

(7) $\displaystyle\int x^2 \left[e^{(x^3)} + \frac{2}{2+x^3} \right] dx$

$$= \int \left[x^2 e^{(x^3)} + \frac{2x^2}{2+x^3} \right] dx$$

$$= \underbrace{\int x^2 e^{(x^3)}\, dx}_{\text{Call this } I_1} + \underbrace{\int \frac{2x^2}{2+x^3}\, dx}_{\text{Call this } I_2}$$

$$I_1 = \int x^2 e^{(x^3)}\, dx$$

Let $u = x^3$
$du = 3x^2\, dx$
$\frac{du}{3} = x^2\, dx$

$$= \frac{1}{3} \int e^u\, du$$

$$= \frac{1}{3} e^u + C_1$$

$$= \frac{1}{3} e^{(x^3)} + C_1$$

$$I_2 = \int \frac{2x^2}{2+x^3}\, dx$$

Let $u = 2 + x^3$
$du = 3x^2\, dx$
$\frac{du}{3} = x^2\, dx$

$$= \frac{2}{3} \int \frac{du}{u}$$

$$= \frac{2}{3} \ln|u| + C_2$$

$$= \frac{2}{3} \ln|2+x^3| + C_2$$

Integral $= I_1 + I_2$

$$= \frac{1}{3} e^{(x^3)} + \frac{2}{3} \ln|2+x^3| + C_1 + C_2$$

$$= \frac{1}{3} e^{(x^3)} + \frac{2}{3} \ln|2+x^3| + C$$

⑧ $\int \frac{dx}{\sqrt{6x - x^2}}$

Complete the Square

$$6x - x^2 = -(x^2 - 6x)$$
$$= -[(x-3)^2 + ?]$$
$$= -[(x-3)^2 - 9]$$
$$= 9 - (x-3)^2$$
$$= 3^2 - (x-3)^2$$

$$\int \frac{dx}{\sqrt{6x - x^2}} = \int \frac{dx}{\sqrt{3^2 - (x-3)^2}}$$

$\boxed{\begin{array}{l} \text{Let } u = x - 3 \\ du = dx \end{array}}$

$$= \int \frac{du}{\sqrt{3^2 - u^2}}$$
$$= \sin^{-1}\left(\frac{u}{3}\right) + C$$
$$= \sin^{-1}\left(\frac{x-3}{3}\right) + C$$

9

$$\int \frac{3\,dx}{x\sin(\ln x)}$$

$$= \int \frac{3\csc(\ln x)}{x}\,dx$$

$$\boxed{\begin{aligned} \text{Let } u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}}$$

$$= 3\int \csc u\,du$$

$$= -3\ln|\csc u + \cot u| + C$$

$$= -3\ln|\csc(\ln x) + \cot(\ln x)| + C$$

(10) $$\int \frac{\cosh \sqrt{x}}{\sqrt{x}}\, dx$$

Let $u = \sqrt{x}$

$du = \frac{1}{2} x^{-1/2}\, dx$

$2\,du = \frac{dx}{\sqrt{x}}$

$$= 2\int \cosh u \, du$$

$$= 2 \sinh u + C$$

$$= 2 \sinh \sqrt{x} + C$$