Math 250A Review Problems

1. Evaluate:

$\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3}$

2. Let $f(x) = 3x^4$ and $g(x) = \sec x$. Find the derivative of:

a) $y = f(x)g(x)$

b) $y = \frac{f(x)}{g(x)}$

c) $y = f(g(x))$

d) $y = g(f(x))$

3. Find $\frac{dy}{dx}$:

a) $y = \ln(x\sqrt{x^2 - 9})$

b) $y = \dfrac{e^{(x^3)}}{e^{7x}}$

4. Find y':

$y = 3 \arcsin e^{6x} + \sqrt{1 - e^{12x}}$

5. Find $\frac{dy}{dx}$ given:

$(x^3 + y^3)^2 = 2xy$

6. Find:

a) $\int_0^1 \frac{7x}{\sqrt{5x^2+4}} dx$

b) $\int \frac{7x}{\sqrt{5x^2+4}} dx$

7. Find $\int x^2 [e^{(x^3)} + \frac{2}{2+x^3}] dx$

8. Find $\int \frac{dx}{\sqrt{6x-x^2}}$

9. Find $\int \frac{3\ dx}{x\sin(\ln x)}$

10. Find $\int \frac{\cosh\sqrt{x}}{\sqrt{x}}dx$

11. Find $\int x^2 \ln x\ dx$

12. Find $\int \sec^4\theta \tan^3\theta\ d\theta$

13. Find $\int \frac{dx}{x^2\sqrt{16-x^2}}$

14. Find $\int \frac{13}{(x+2)(x^2+9)}dx$

15. Evaluate:

a) $\lim_{x\to 0} \frac{\sin 6x}{\tan 7x}$

b) $\lim_{x\to 0^+} (e^x + 5x)^{\frac{1}{x}}$

16. Evaluate $\int_0^\infty \frac{e^x}{1+(e^x)^2}\ dx.$

17. Consider each sequence below. Find the first three terms and find the sequence's limit (if it exists).

a) $a_n = \frac{e^n}{n}$ for $1 \le n < \infty$

b) $a_n = \frac{4n}{\sqrt{n^2+1}}$ for $0 \le n < \infty$

18. Find the sum (if it exists).

a) $\sum_{n=2}^{\infty} \frac{7}{(n+3)(n+4)}$

b) $\sum_{n=2}^{\infty} \frac{2^{n+1}}{7^n}$

19. The series $\sum_{n=1}^{\infty} \frac{2}{n^2}$ converges by the Integral Test.
Find N so that $R_N \leq 0.1$.

20. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges by the Alternating Series Test.

a) Find S_3.
b) Find an upper bound for $|R_3|$.
c) Estimate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ using parts a) and b).

21. Decide if the series converges or diverges by using an appropriate test.

a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$

b) $\sum_{n=1}^{\infty} n^2 e^{(-n^3)}$

c) $\sum_{n=1}^{\infty} \frac{n}{2n+1}$

d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$

e) $\sum_{n=1}^{\infty} \frac{1}{3^{(n^3)}}$

f) $\sum_{n=1}^{\infty} \frac{\sqrt{n}+3}{7n^2+2}$

g) $\sum_{n=1}^{\infty} \frac{n!}{(1)(3)(5)\cdots(2n-1)}$

h) $\sum_{n=1}^{\infty} \frac{1}{(1+\ln n)^n}$

22. Find N so that the Maclaurin polynomial $P_N(x)$ approximates $e^{-0.2}$ with $|\text{error}| < 0.0001$.

23. Find the interval of convergence:
$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2 \cdot 2^n}$

24. Find the first four nonzero terms of the Maclaurin series of $f(x) = xe^{-2x}$.

25. Use three nonzero terms of an appropriate series to approximate: $\int_0^{0.5} \sqrt[3]{1+x^2}\, dx$.

26. Eliminate the parameter:
$x = 2t, \quad y = 4t - 12t^2$

27. Find all points where the tangent line is horizontal or vertical:
$x = 4\cos^2 t, \quad y = 2\sin t, \quad 0 \le t \le \frac{\pi}{2}$.

28. Find the length of the curve given by:
$x = \cos t, \quad y = t - \sin t, \quad 0 \le t \le \pi$.

29. Let $r = 1 + \sin\theta$. Find all values of θ where the tangent is horizontal or vertical.

30. Find the area inside both $r = 3 - 2\sin\theta$ and $r = -3 + 2\sin\theta$.

31. a) Find $\frac{d}{dt}\, 9t[t^2, t^3]$ in two different ways.

b) Let $\vec{r}(t) = [7t + 1, 4t]$. Find $\frac{d}{dt}\, \vec{r}(2t)$ in two different ways.

c) Let $\vec{r}(t) = [t^3, 7t, t^2]$. Find $\vec{r}\,'(t) \times \vec{r}\,''(t)$.

d) Find $\int_1^3 [6t^2, 8t]\, dt$.

32. A particle has $\vec{a}(t) = [4, 0, -9.8], \vec{r}(0) = [0, 0, 0]$, and $\vec{v}(0) = [0, 25, 25]$. Find its speed when it hits the ground.

33. Given $\vec{r}(t) = [t, 1 + \frac{1}{t}, 0]$. Find the tangential and normal components of the acceleration.

34. Find the arc length of the curve described by:

$\vec{r}(t) = [\cos^3 t, \sin^3 t]$, $0 \le t \le \frac{\pi}{2}$.