(26)

$x - 2 = 4\cos t$

$\frac{x-2}{4} = \cos t$

$y + 1 = 3\sin t$

$\frac{y+1}{3} = \sin t$

$$\cos^2 t + \sin^2 t = 1$$

$$\left(\frac{x-2}{4}\right)^2 + \left(\frac{y+1}{3}\right)^2 = 1$$

or $$\frac{(x-2)^2}{16} + \frac{(y+1)^2}{9} = 1$$

(27) $\frac{dx}{dt} = 2e^{2t} - 2$ $\qquad \frac{dy}{dt} = \frac{2}{t+1} - 1$

Set $\frac{dx}{dt} = 0$: $\quad 2e^{2t} - 2 = 0$

$$2e^{2t} = 2$$
$$e^{2t} = 1$$
$$2t = 0$$
$$t = 0$$

Set $\frac{dy}{dt} = 0$: $\quad \frac{2}{t+1} - 1 = 0$

$$\frac{2}{t+1} = 1$$
$$2 = t+1$$
$$t = 1$$

Horizontal Tangent when $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$

$$t = 1$$
$$(x,y) = (e^2 - 2,\ 2\ln 2 - 1)$$

Vertical Tangent when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$

$$t = 0$$
$$(x,y) = (1, 0)$$

(28) $\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = t^{-1/2}$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{1 + \frac{1}{t}}$$

$$S_x = 2\pi \int_a^b y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= 2\pi \int_0^4 2\sqrt{t} \sqrt{1 + \frac{1}{t}}\, dt$$

$$= 4\pi \int_0^4 \sqrt{t+1}\, dt$$

$$\boxed{\begin{array}{l} \text{Let } u = t+1 \\ \quad du = dt \\ t = 0 \Rightarrow u = 1 \\ t = 4 \Rightarrow u = 5 \end{array}}$$

$$= 4\pi \int_1^5 \sqrt{u}\, du$$

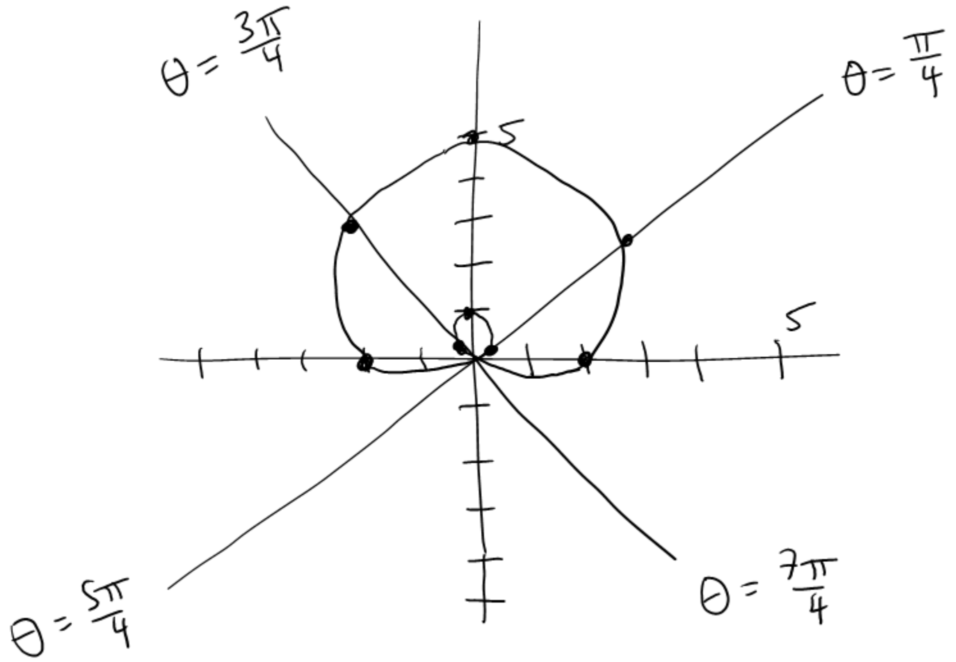
$$= 4\pi \left[\frac{2}{3} u^{3/2} \right]_1^5$$

$$= \frac{8\pi}{3} \left(5^{3/2} - 1 \right)$$

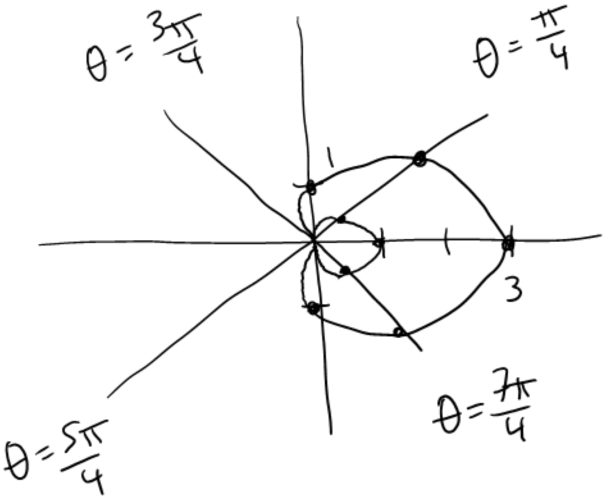
(29)

θ	$r = 2 + 3\sin\theta$
0	2
$\frac{\pi}{4}$	4.1
$\frac{2\pi}{4}$	5
$\frac{3\pi}{4}$	4.1

θ	$r = 2 + 3\sin\theta$
$\frac{4\pi}{4}$	2
$\frac{5\pi}{4}$	-0.1
$\frac{6\pi}{4}$	-1
$\frac{7\pi}{4}$	-0.1



(30) $r = 1 + 2\cos\theta$



θ	r
0	3
$\frac{\pi}{4}$	2.4
$\frac{2\pi}{4}$	1
$\frac{3\pi}{4}$	-0.4

θ	r
$\frac{4\pi}{4}$	-1
$\frac{5\pi}{4}$	-0.4
$\frac{6\pi}{4}$	1
$\frac{7\pi}{4}$	2.4

Set $r = 0$

$$1 + 2\cos\theta = 0$$

$$2\cos\theta = -1$$

$$\cos\theta = \frac{-1}{2}$$

$$\boxed{\cos^{-1}\frac{1}{2} = \frac{\pi}{3}}$$

S	A
T	C

$$\theta = \frac{2\pi}{3},\ \frac{4\pi}{3}$$

$$A = \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1 + 2\cos\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1 + 4\cos\theta + 4\cos^2\theta)\, d\theta$$

$$= \frac{1}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \left(1 + 4\cos\theta + 2 + 2\cos 2\theta\right) d\theta$$

$$= \frac{1}{2} \left[3\theta + 4\sin\theta + \sin 2\theta \right]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}$$

$$= \frac{1}{2} \left[\left(4\pi + 4\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2} \right) - \left(2\pi + 4\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{2} \right) \right]$$

$$= \frac{1}{2} \left[2\pi - 3\sqrt{3} \right]$$

$$= \pi - \frac{3\sqrt{3}}{2}$$

(31) a) $\frac{d}{dt} t^3 [t^2, 2t] = \frac{d}{dt} [t^5, 2t^4]$

$= [5t^4, 8t^3]$

$\frac{d}{dt} t^3 [t^2, 2t] = t^3 [2t, 2] + 3t^2 [t^2, 2t]$

$= [2t^4, 2t^3] + [3t^4, 6t^3]$

$= [5t^4, 8t^3]$

b) $\vec{r}(3t) = [24t, 27t^3 + 1]$

$\frac{d}{dt} \vec{r}(3t) = [24, 81t^2]$

$\vec{r}\,'(t) = [8, 3t^2]$

$\vec{r}\,'(3t) = [8, 27t^2]$

$\frac{d}{dt} \vec{r}(3t) = \vec{r}\,'(3t)\ (3)$

$= [8, 27t^2]\ (3)$

$= [24, 81t^2]$

c) $\vec{r}\,'(t) = [4t, 9t^2]$

$\vec{r}\,''(t) = [4, 18t]$

$\vec{r}\,'(t) \cdot \vec{r}\,''(t) = 16t + 162t^3$

d) $\vec{r}'(t) = [4t^3, 2e^{2t}]$

$\vec{r}(t) = [t^4, e^{2t}] + \vec{c}$

Sub $t=0$: $\vec{r}(0) = [0, 1] + \vec{c}$

$[1, 3] = [0, 1] + \vec{c}$

$\vec{c} = [1, 2]$

$\vec{r}(t) = [t^4, e^{2t}] + [1, 2]$

$= [t^4+1, e^{2t}+2]$

32 $y_0 = 1 \quad v_0 = 40 \quad \theta = 30° \quad g = 9.8$

$\vec{r} = [(v_0 \cos\theta)t, y_0 + (v_0 \sin\theta)t - \frac{gt^2}{2}]$

$= [20\sqrt{3}t, 1 + 20t - 4.9t^2]$

$\vec{v} = [20\sqrt{3}, 20 - 9.8t]$

Max height occurs when

(y-component of $\vec{v}$) $= 0$

$20 - 9.8t = 0$

$t = \frac{20}{9.8}$

Max height is (y-component of $\vec{r}$) $\Big|_{t=\frac{20}{9.8}}$

$= (1 + 20t - 4.9t^2)\Big|_{t=\frac{20}{9.8}}$

≈ 21.4 m

(33)

$\bar{r} = [a\cos wt, a\sin wt, 0]$

$\bar{v} = [-aw\sin wt, aw\cos wt, 0]$

$\vec{a} = [-aw^2\cos wt, -aw^2\sin wt, 0]$

$\bar{v}\cdot\bar{a} = a^2w^3\cos wt\sin wt - a^2w^3\cos wt\sin wt$

$= 0$

$a_T = \dfrac{\bar{v}\cdot\bar{a}}{\|\bar{v}\|}$

$= 0$

$$\bar{v}\times\bar{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -aw\sin wt & aw\cos wt & 0 \\ -aw^2\cos wt & -aw^2\sin wt & 0 \end{vmatrix}$$

$= \vec{i}(0) - \vec{j}(0)$

$+\vec{k}(a^2w^3\sin^2 wt + a^2w^3\cos^2 wt)$

$= a^2w^3(\sin^2 wt + \cos^2 wt)\vec{k}$

$= a^2w^3\vec{k}$

$\|\bar{v}\times\bar{a}\| = \sqrt{(a^2w^3)^2}$

$= |a^2w^3|$

$= a^2w^3$

$\|\bar{v}\| = \sqrt{a^2w^2\sin^2 wt + a^2w^2\cos^2 wt}$ $\rightarrow$

$$= \sqrt{(a^2w^2)(\sin^2 wt + \cos^2 wt)}$$

$$= \sqrt{a^2w^2}$$

$$= |aw|$$

$$= aw$$

$$a_N = \frac{\| \bar{v} \times \bar{a} \|}{\| \bar{v} \|}$$

$$= \frac{a^2w^3}{aw}$$

$$= aw^2$$

(34) $\bar{r} = [1+3t, \cos 2t, \sin 2t]$

$\bar{v} = [3, -2\sin 2t, 2\cos 2t]$

$$\|\bar{v}\| = \sqrt{9 + 4\sin^2 2t + 4\cos^2 2t}$$

$$= \sqrt{9 + 4(\sin^2 2t + \cos^2 2t)}$$

$$= \sqrt{9+4}$$

$$= \sqrt{13}$$

$$s = \int_0^{4\pi} \sqrt{13}\, dt$$

$$= \sqrt{13}\, t \Big|_0^{4\pi}$$

$$= 4\pi\sqrt{13}$$