(1)
$$\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$$

$$= \lim_{x \to 0} \frac{(\sqrt{2+x} - \sqrt{2})}{x} \cdot \frac{(\sqrt{2+x} + \sqrt{2})}{(\sqrt{2+x} + \sqrt{2})}$$

$$= \lim_{x \to 0} \frac{2+x-2}{x(\sqrt{2+x} + \sqrt{2})}$$

$$= \lim_{x \to 0} \frac{x}{x(\sqrt{2+x} + \sqrt{2})}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{2+x} + \sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$

2 a) $y = 2x^3 \csc x$

$y' = -2x^3 \csc x \cot x + 6x^2 \csc x$

b) $y = \dfrac{2x^3}{\csc x}$

$y' = \dfrac{(\csc x)(6x^2) - 2x^3(-\csc x \cot x)}{\csc^2 x}$

$= \dfrac{6x^2 \csc x + 2x^3 \csc x \cot x}{\csc^2 x}$

or $y' = \dfrac{6x^2 + 2x^3 \cot x}{\csc x}$

c) $y = 2[\csc x]^3$

$y' = 6[\csc x]^2 (-\csc x \cot x)$

$= -6 \csc^3 x \cot x$

d) $y = \csc(2x^3)$

$y' = -\csc(2x^3) \cot(2x^3)\ (6x^2)$

$= -6x^2 \csc(2x^3) \cot(2x^3)$

3 a) $y = \ln\left(\dfrac{\sqrt{5+x^2}}{x}\right)$

$$= \ln\sqrt{5+x^2} - \ln x$$

$$= \frac{1}{2}\ln(5+x^2) - \ln x$$

$$\frac{dy}{dx} = \frac{1}{2} \quad \frac{2x}{5+x^2} - \frac{1}{x}$$

$$= \frac{x}{5+x^2} - \frac{1}{x}$$

b) $y = e^{x^2+4x}$

$$\frac{dy}{dx} = (2x+4)\,e^{x^2+4x}$$

(4) $y = 4x \arctan x - \ln(1+x^2)^2$

$= 4x \arctan x - 2\ln(1+x^2)$

$y' = 4x\left(\frac{1}{1+x^2}\right) + 4\arctan x - 2 \cdot \frac{2x}{1+x^2}$

$= 4\arctan x$

5 $(x^2+y^2)^2 = 2x^2y$

$$2(x^2+y^2)[2x+2yy'] = 2x^2y' + 4xy$$

$$4x(x^2+y^2) + 4y(x^2+y^2)y' = 2x^2y' + 4xy$$

$$4y(x^2+y^2)y' - 2x^2y' = 4xy - 4x(x^2+y^2)$$

$$[4y(x^2+y^2) - 2x^2]y' = 4xy - 4x(x^2+y^2)$$

$$y' = \frac{4xy - 4x(x^2+y^2)}{4y(x^2+y^2) - 2x^2}$$

or $$y' = \frac{2xy - 2x^3 - 2xy^2}{2x^2y + 2y^3 - x^2}$$

6 a) $$\int_0^4 \frac{x}{(x^2+1)^3}\,dx$$

$$\boxed{\begin{aligned} &\text{Let } u = x^2+1 \\ &du = 2x\,dx \\ &\frac{du}{2} = x\,dx \\ &x=0 \Rightarrow u=1 \\ &x=4 \Rightarrow u=17 \end{aligned}}$$

$$= \frac{1}{2}\int_1^{17} \frac{du}{u^3}$$

$$= \frac{1}{2}\int_1^{17} u^{-3}\,du$$

$$= \frac{1}{2}\left[-\frac{1}{2}u^{-2}\right]_1^{17}$$

$$= -\frac{1}{4}\left(\frac{1}{289} - 1\right)$$

$$= \frac{72}{289}$$

b) $$\int \frac{x}{(x^2+1)^3}\,dx$$

$$\boxed{\begin{aligned} &\text{Let } u = x^2+1 \\ &du = 2x\,dx \\ &\frac{du}{2} = x\,dx \end{aligned}}$$

$$= \frac{1}{2}\int \frac{du}{u^3}$$

$$= \frac{1}{2}\int u^{-3}\,du$$

$$= \frac{1}{2}\left[-\frac{1}{2}u^{-2}\right] + C$$

$$= -\frac{1}{4}(x^2+1)^{-2} + C$$

$$= \frac{-1}{4(x^2+1)^2} + C$$

(7) $\displaystyle\int \frac{7e^{4x}}{3+6e^{4x}}\,dx$

$$\boxed{\begin{aligned} \text{Let } u &= 3+6e^{4x} \\ du &= 24e^{4x}\,dx \\ \frac{du}{24} &= e^{4x}\,dx \end{aligned}}$$

$$= \frac{7}{24}\int \frac{du}{u}$$

$$= \frac{7}{24}\ln|u| + C$$

$$= \frac{7}{24}\ln\left|3+6e^{4x}\right| + C$$

(8) $\displaystyle\int \frac{2x+5}{x^2+4x+13}\,dx$

Complete the Square

$$x^2+4x+13 = (x+2)^2 + ?$$
$$= (x+2)^2 + 9$$
$$= (x+2)^2 + 3^2$$

$$2x+5 = ?\,(x+2) + ?$$
$$= 2(x+2) + ?$$
$$= 2(x+2) + 1$$

$$\int \frac{2x+5}{x^2+4x+13}\,dx = \int \frac{2(x+2)+1}{(x+2)^2+3^2}\,dx$$

Let $u = x+2$
$du = dx$

$$= \int \frac{2u+1}{u^2+3^2}\,du$$
$$= \int \frac{2u}{u^2+3^2}\,du + \int \frac{1}{u^2+3^2}\,du$$
$$= \ln|u^2+3^2| + \frac{1}{3}\tan^{-1}\frac{u}{3} + C$$
$$= \ln|(x+2)^2+3^2| + \frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + C$$

or $\displaystyle \ln|x^2+4x+13| + \frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + C$

(9) $\int e^{2x} \csc e^{2x} \cot e^{2x} \, dx$

$$\boxed{\begin{aligned} \text{Let } u &= e^{2x} \\ du &= 2e^{2x} dx \\ \frac{du}{2} &= e^{2x} dx \end{aligned}}$$

$$= \frac{1}{2} \int \csc u \cot u \, du$$

$$= -\frac{1}{2} \csc u + C$$

$$= -\frac{1}{2} \csc e^{2x} + C$$

⑩ $\int (t^2+4t^3)\sinh(t^3+3t^4)\,dt$

Let $u = t^3+3t^4$
$du = (3t^2+12t^3)\,dt$
$\frac{du}{3} = (t^2+4t^3)\,dt$

$= \frac{1}{3}\int \sinh u \, du$

$= \frac{1}{3}\cosh u + C$

$= \frac{1}{3}\cosh(t^3+3t^4) + C$