Math 250A Practice Problems

1. Evaluate:

$\lim_{x \to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$

2. Let $f(x) = 2x^3$ and $g(x) = \csc x$. Find the derivative of:

a) $y = f(x)g(x)$

b) $y = \frac{f(x)}{g(x)}$

c) $y = f(g(x))$

d) $y = g(f(x))$

3. Find $\frac{dy}{dx}$:

a) $y = \ln(\frac{\sqrt{5+x^2}}{x})$

b) $y = e^{(x^2)}e^{4x}$

4. Find y':

$y = 4x \arctan x - \ln(1 + x^2)^2$

5. Find $\frac{dy}{dx}$ given:

$(x^2 + y^2)^2 = 2x^2y$

6. Find:

a) $\int_0^4 \frac{x}{(x^2+1)^3} dx$

b) $\int \frac{x}{(x^2+1)^3} dx$

7. Find $\int \frac{7e^{4x}}{3+6e^{4x}} dx$

8. Find $\int \frac{2x+5}{x^2+4x+13} dx$

9. Find $\int e^{2x} \csc e^{2x} \cot e^{2x} dx$

10. Find $\int (t^2 + 4t^3) \sinh(t^3 + 3t^4) dt$

11. Find $\int \arctan x \; dx$

12. Find $\int \sin^3 \theta \; d\theta$

13. Find $\int \frac{dx}{\sqrt{9x^2+1}}$

14. Find $\int \frac{2x}{(x+1)(x^2+4)} dx$

15. Evaluate:

a) $\lim_{x \to 0} \frac{\tan 2x}{\ln(1+x)}$

b) $\lim_{x \to \infty} (1 - \frac{1}{x})^{2x}$

16. Evaluate $\int_0^\infty x e^{-x} \; dx$.

17. Consider each sequence below. Find the first three terms and find the sequence's limit (if it exists).

a) $a_n = (-2)^n$ for $1 \leq n < \infty$

b) $a_n = \frac{3n}{4n+1}$ for $2 \leq n < \infty$

18. Find the sum (if it exists).

a) $\sum_{n=5}^{\infty} \frac{2}{(n+1)(n+2)}$

b) $\sum_{n=1}^{\infty} \frac{4(7^{n-1})}{8^n}$

19. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the Integral Test.

a) Find S_4.
b) Find an upper bound for R_4.
c) Estimate $\sum_{n=1}^{\infty} \frac{1}{n^2}$ using parts a) and b).

20. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges by the Alternating Series Test.
Find N so that $|R_N| \leq 0.01$.

21. Decide if the series converges or diverges by using an appropriate test.

a) $\sum_{n=1}^{\infty} \frac{1}{n^{0.9}}$

b) $\sum_{n=1}^{\infty} ne^{(-n^2)}$

c) $\sum_{n=0}^{\infty} \frac{n+1}{n+2}$

d) $\sum_{n=1}^{\infty} \frac{1+|\cos n|}{n}$

e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

f) $\sum_{n=1}^{\infty} (\frac{3n+4}{4n+3})^n$

g) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

h) $\sum_{n=1}^{\infty} \frac{\sqrt{n}+3}{7n+2}$

22. a) Find the 4th degree Taylor polynomial of
$f(x) = \cos x$ centred at $c = \frac{\pi}{2}$.
b) Use it to approximate $\cos 1.5$.
c) Find an upper bound for $|R_4(1.5)|$.
d) Approximate $\cos 1.5$, with error.

23. Find the interval of convergence:
$\sum_{n=1}^{\infty} \frac{(x-4)^n}{n \cdot 6^n}$

24. Find the first four nonzero terms of the Maclaurin series of $f(x) = \sqrt[3]{1+x^2}$.

25. Use three nonzero terms of an appropriate series to approximate:
$\int_0^1 \cos x^2 \, dx$

26. Eliminate the parameter:
$x = 2 + 4\cos t, \quad y = -1 + 3\sin t$

27. Find all points where the tangent line is horizontal or vertical:
$x = e^{2t} - 2t, \quad y = 2\ln(t+1) - t$

28. The following curve is revolved about the x-axis to produce a solid. Find the surface area of the solid:
$x = t + 1, \quad y = 2\sqrt{t}, \quad 0 \le t \le 4.$

29. Graph $r = 2 + 3\sin\theta$.

30. Find the area of the inner loop of $r = 1 + 2\cos\theta$.

31. a) Find $\frac{d}{dt}\, t^3[t^2, 2t]$ in two different ways.

b) Let $\vec{r}(t) = [8t, t^3 + 1]$. Find $\frac{d}{dt}\, \vec{r}(3t)$ in two different ways.

c) Let $\vec{r}(t) = [2t^2, 3t^3]$. Find $\vec{r}\,'(t) \cdot \vec{r}\,''(t)$.

d) Find $\vec{r}(t)$ if $\vec{r}\,'(t) = [4t^3, 2e^{2t}]$ and $\vec{r}(0) = [1, 3]$.

32. A projectile is launched with an initial height of 1 m, an initial velocity of 40 m/s, and an angle of inclination of 30°. Find its maximum height.

33. Let a and ω be positive constants. Given $\vec{r}(t) = [a\cos\omega t, a\sin\omega t, 0]$. Find the tangential and normal components of the acceleration.

34. Find the arc length of the curve described by:

$\vec{r}(t) = [1 + 3t, \cos 2t, \sin 2t]$, $0 \le t \le 4\pi$.